A Fast Minimum-Chi-Square Estimation of Thailand's Daily Real Yields

Anya Khanthavit*

Abstract

Identifying necessary parameters using the linear projection approach in a latent multifactor interest model can be numerical challenging due to highly nonlinear and badly behaved objective surfaces. This study applies the minimum-chi-square estimation uccessfully in order to lessen computation time for Thailand's daily real yields. Using monthly headline inflation rates from July 2001 to May 2014 and daily up-to-15-year nominal yield curves from July 2, 2001 to May 30, 2014, it finds a normal shape for the average daily real curve. The average 1-month real yield is -0.5446%. The yields are rising with maturities. The average 10- and 15-year yields are 3.1215% and 3.9199%. The average curve is higher than those reported earlier, for which its short yields are about 10 basis points higher and its long yields are about 40 basis points higher. The resulting curves are useful and practical because they cover longer maturities and consume less computation time.

Keywords : Daily Real Yields, Affine Mutifactor Interest Rate Model, Daily Real-Yield Estimation, Minimum-Chi-Square Estimation

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การกำหนดอัตราดอกเบี้ยที่แท้จริงของประเทศไทยเป็นรายวัน โดยวิธี Minimum Chi-Square เพื่อการประมวลผลที่รวดเร็ว

อัญญา ขันธวิทย์*

บทคัดย่อ

เนื่องจากผิวพื้นของฟังก์ชันวัตถุประสงค์มีลักษณะไม่เป็นเส้นตรงและมีพฤติกรรมไม่สม่ำเสมอ การระบุค่าพารามิเตอร์สำคัญในตัวแบบจำลองเพื่อกำหนดอัตราดอกเบี้ยโดยอ้างอิงเชิงเส้นตรงกับกลุ่มปัจจัยแฝงจึงต้องใช้เวลายาวนานเพื่อคำนวณ การศึกษาประยุกต์ใช้วิธี Minimum Chi-Square ในการกำหนดค่าพารามิเตอร์ของตัวแบบได้สำเร็จและสามารถลดเวลาประมวลผลลงได้มาก เมื่อการศึกษาใช้ข้อมูลอัตราเงินเฟ้อรายเดือน ตั้งแต่เดือนกรกฎาคม 2544 ถึงเดือนพฤษภาคม 2557 และข้อมูลรายวันของโครงสร้างอัตราดอกเบี้ยรูปตัวเงินระยะยาวนานถึง 15 ปี ตั้งแต่วันที่ 2 กรกฎาคม 2544 ถึงวันที่ 30 พฤษภาคม 2557 การศึกษาพบว่า โครงสร้างอัตราดอกเบี้ยที่แท้จริงรายวันเฉลี่ยมีรูปทรงปกติ อัตราดอกเบี้ยที่แท้จริงระยะ 1 เดือนเฉลี่ยมีระดับเท่ากับ -0.5446% ระดับอัตราดอกเบี้ยเพิ่มขึ้นตามระยะการลงทุน อัตราสำหรับระยะ 10 ปี และ 15 ปี มีระดับเท่ากับ 3.1215% และ 3.9199% โครงสร้างอัตราดอกเบี้ยที่แท้จริงเฉลี่ยมีระดับสูงกว่าที่รายงานไปก่อนหน้าโดยการศึกษาอื่น โดยที่อัตราดอกเบี้ยระยะสั้นมีระดับสูงกว่าประมาณ 10 จุดเบซิส และระยะยาวประมาณ 40 จุดเบซิส โครงสร้างอัตราดอกเบี้ยที่ได้เป็นผลลัพธ์มีประโยชน์และสามารถนำไปปฏิบัติได้ เนื่องจากโครงสร้างครอบคลุมระยะของดอกเบี้ยที่ยาวนานกว่า ในขณะที่ใช้เวลาคำนวณที่น้อยกว่ามาก

คำหลัก : อัตราดอกเบี้ยที่แท้จริงรายวัน ตัวแบบจำลองเพื่อกำหนดอัตราดอกเบี้ยโดยอ้างอิงเชิงเส้นตรงกับกลุ่มปัจจัยแฝง การกำหนดอัตราดอกเบี้ยที่แท้จริงเป็นรายวัน วิธี Minimum Chi-Square

* ผู้วิจัยขอบคุณคณะพาณิชยศาสตร์และการบัญชี มหาวิทยาลัยธรรมศาสตร์ ที่สนับสนุนทุนวิจัยและขอบคุณสมาคมตลาดตราสารหนี้ไทยที่สนับสนุนข้อมูลโครงสร้างอัตราดอกเบี้ย การติดต่อผู้เขียนสามารถทำได้ที่ อัญญา ขันธวิทย์ คณะพาณิชยศาสตร์และการบัญชี มหาวิทยาลัยธรรมศาสตร์ ท่าพระจันทร์ กรุงเทพฯ 10200 ที่อยู่อิเล็กทรอนิกส์ akhantha@tu.ac.th

1. Introduction

Daily estimates of real yields are useful and important. They support more active trading of the securities--especially inflation-linked bonds, and closer monitoring of the economy. Noticing that previous studies could give only monthly or bi-weekly estimates, Khanthavit (2014a, 2014b) proposed a linear projection approach to estimate real yields on a daily basis. The approach employs monthly inflation and daily nominal yield data. It is useful particularly for emerging markets because in general these two series are the only available datasets. However, as was pointed out by Hamilton and Wu (2012) and experienced by Khanthavit (2014a, 2014b), the estimation is numerical challenging due to highly nonlinear and badly behaved objective surfaces.

This study is a note to Khanthavit (2014a, 2014b) which applies Rothenberg's (1973) minimum-chi-square estimation technique successfully in order to lessen computation time for Thailand's daily real yields. It contributes to the literature by extending Khanthavit (2014a, 2014b) into at least three important ways. Firstly, it derives an exact functional relationship between the monthly and daily AR(1) parameters for inflation. The functional relationship is important because it allows the econometrician to estimate the daily AR(1) parameters directly from monthly inflation data. Secondly, it employs longer nominal-yield curves of up to 15 years, as opposed to those of up to 10 years in the previous studies. The use of up-to-15-year curves is more consistent with the durations of inflation-linked bonds currently traded on the Thai bond market.[1] Thirdly, its sample period is more complete, covering from the day the Thai Bond Market Association reported the first nominal curve of up to 15-year duration to a much more recent date.

Using monthly headline inflation rates from July 2001 to May 2014 and daily up-to-15-year nominal curves from July 2, 2001 to May 30, 2014, the study finds a normal shape for the average daily real curve. The average 1-month real yield is -0.5446%. The yields are rising with maturities. The average 10- and 15-year yields are 3.1215% and 3.9199%. The average curve is higher than those reported earlier for Thailand, for which its short yields are about 10 basis points higher and its long yields are about 40 basis points higher. The resulting curves are useful and practical because they cover maturities of up to 15 years and consume computation time of less than one thirtieth of that in previous studies.

[1] ILB217A and ILB283A are the two issues on the market. Their issue terms are 10 and 15 years and, as of August 2014, their times to maturity are 6.97 and 13.63 years.

2. The Model

This study is a note to Khanthavit (2014a, 2014b), in which the model of Joyce et al. (2010) is adopted to describe nominal and real yields in Thailand. The model is an essentially affine term structure model which relates the nominal and real yields with a set of latent factors linearly under a no-arbitrage condition in the real world. It is flexible for it allows time-varying risk premiums and real short rate. The number of latent factors can be raised to capture complex behavior of the yields. Moreover, a latent factor model is found in previous studies to fit yields better than a macro factor model.

2.1 The Pricing of Real and Nominal Bonds

In a no-arbitrage environment, the time-t price $P_t^{n,R}$ of a zero-coupon real bond with an n-period maturity must be given by (Cochrane (2005))

$$P_t^{n,R} = E_t\{M_{t+1} M_{t+2} \dots M_{t+n}\}, \tag{1}$$

where M_{t+j} is the real pricing kernel in j periods hence and $E_t\{.\}$ is the conditional expectation operator in the real world. The price $P_t^{n,N}$ of a zero-coupon nominal bond is given in a similar way but with the nominal pricing kernel $M^*_{t+j} = M_{t+j}\frac{I_{t+j-1}}{I_{t+j}}$ being substituted for M_{t+j} I_{t+j}. is the consumer price index at time t+j.

$$P_t^{n,N} = E_t\{M^*_{t+1} M^*_{t+2} \dots M^*_{t+n}\}. \tag{2}$$

2.2 Real Yields and Nominal Yields

From eqs. (1) and (2), because the real yield $y_t^{n,R}$ and nominal yield $y_t^{n,N}$ are $-\frac{1}{n}Ln\{P_t^{n,R}\}$ and $-\frac{1}{n}Ln\{P_t^{n,N}\}$, up to a second order approximation the yields must equal

$$y_t^{n,R} = -\frac{1}{n}\left\{E_t\left(\sum_{j=1}^{n} m_{t+j}\right) + \frac{1}{2}V_t\left(\sum_{j=1}^{n} m_{t+j}\right)\right\} \tag{3.1}$$

$$y_t^{n,N} = -\frac{1}{n}\left\{E_t\left(\sum_{j=1}^{n}\left(m_{t+j} - \pi_{t+j}\right)\right) + \frac{1}{2}V_t\left(\sum_{j=1}^{n}\left(m_{t+j} - \pi_{t+j}\right)\right)\right\} \tag{3.2}$$

where $m_{t+j} = Ln\{m_{t+j}\}$. $\pi_{t+j} = Ln\left\{\frac{I_{t+j-1}}{I_{t+j}}\right\}$ is logged inflation. V_t (.) is the variance operator conditioned on the information at time t.

2.3 Stochastic Behavior of Pricing Kernels

The logged, real pricing kernel m_{t+1} takes on the form as in eq. (4).

$$m_{t+j} = -(\bar{r} + \gamma^{T} z_t) - \frac{\Lambda_t' \Omega \Lambda_t}{2} - \Lambda_t' \Omega^{\frac{1}{2}} \varepsilon_{t+1} \qquad (4)$$

The term $(\bar{r} + \gamma' z_t)$ is the real short rate. It can vary over time with a set of K latent factors $z_t' = [z_{1,t}, ..., z_{k,t}]$. The real short rate is constant if $\gamma' = [\gamma_1, ..., \gamma_k]$ is a zero vector. Vector $\Lambda_t' \Omega^{\frac{1}{2}}$ is time-varying risk premiums.

$$\Lambda_t = \lambda + \beta z_t. \qquad (5)$$

Vector $\lambda' = [\lambda_1, ..., \lambda_K]$ and matrix $\beta = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1K} \\ \vdots & \ddots & \vdots \\ \beta_{K1} & \cdots & \beta_{KK} \end{bmatrix}$. The risk premium for factor k is constant if vector $[\beta_{k1}, ..., \beta_{kk})$ is zero. $\varepsilon_{t+1}' = [\varepsilon_{1,t+1}, ..., \varepsilon_{K,t+1}]$ are Gaussian shocks of factors z_{t+1}. Their mean vector is zero and their covariance matrix is

$\Omega = \begin{bmatrix} \sigma_1^2 & 0 \cdots & 0 \\ 0 & \ddots & \vdots \\ \vdots & & 0 \\ 0 & \cdots 0 & \sigma_K^2 \end{bmatrix}$. Factors z_{t+1} follow a VAR(1) process in eq. (6).

$$z_{t+1} = \varphi z_t + \varepsilon_{t+1}. \qquad (6)$$

Coefficient matrix $\varphi = \begin{bmatrix} \varphi_{11} & 0 \cdots & & 0 \\ \varphi_{21} & \varphi_{22} & 0 \cdots & \vdots \\ \vdots & & \ddots & 0 \\ \varphi_{K1} & \varphi_{K2} & \cdots & \varphi_{KK} \end{bmatrix}$ is a lower triangular matrix.

Because the logged nominal pricing kernel M_{t+1}^* is $M_{t+1} - \pi_{t+1}$, from eq. (4), it must equal

$$M_{t+1}^* = -(\bar{r} + \gamma' z_t) - \frac{\Lambda_t' \Omega \Lambda_t}{2} \Lambda_t' \Omega^{\frac{1}{2}} \varepsilon_{t+1} - \pi_{t+1}. \qquad (7)$$

2.4 The Pricing

Following Duffie and Kan (1996), Joyce et al. (2010) derived the solutions for the real and nominal yields as affine functions of latent factors in eqs. (8) and (9).

$$y_t^{n,R} = -\frac{1}{n}\{A_n + B_n' z_t\} \tag{8}$$

$$y_t^{n,N} = -\frac{1}{n}\{A_n^* + B_n^{*\prime} z_t\}, \tag{9}$$

where $A_0 = A_0^* = 0.00$ and $B_0 = B_0^*$ are (Kx1) zero vectors. Coefficients $A_{n>0}$ and $A_{n>0}^*$ and vectors $B_{n>0}$ and $B_{n>0}^*$ are determined sequentially with respect to the systems of equations (10).

$$A_n = -\bar{r} + A_{n-1} - B_{n-1}' \Omega\lambda + \frac{1}{2} B_{n-1}' \Omega B_{n-1} \tag{10.1}$$

$$B_n' = -\gamma' + B_{n-1}' (\varphi - \Omega\beta) \tag{10.2}$$

and

$$A_n^* = -\bar{r} - \mu_\pi + A_{n-1}^* - B_n^{*\prime}\Omega\lambda^* + \frac{1}{2} B_{n-1}^{*\prime}\Omega B_{n-1}^* + \frac{\sigma_1^2}{2} + \sigma_1^2\lambda_1 \tag{10.3}$$

$$B_n^{*\prime} = -(\gamma + \varphi_1) + B_{n-1}^{*\prime}(\varphi - \Omega\beta) + \iota'\Omega\beta. \tag{10.4}$$

where $\varphi_1 = [\varphi_{11}\ 0 \ldots 0]$, $\iota' = [1\ \ 0 \ldots 0]$ and $\lambda^* = \lambda + \iota$. μ_π is the unconditional mean of the inflation. The specifications (10.3) and (10.4) are specific to the perfect correlation assumption of factor $z_{1,t}$ with inflation π_t. Modification needs be made under a different assumption for π_t.

3. Model Estimation

3.1 Measurement Equations

Because factors z_t are latent, the econometrician will have to relate them with observed variables. Khanthavit (2014a, 2014b) considers inflation and nominal yields because these variables are observed in most countries. The measurement equations for *day t* are given by

$$\begin{bmatrix} \pi_t \\ -n_1 y_t^{n_1,N} \\ \vdots \\ -n_H y_t^{n_H,N} \end{bmatrix} = \begin{bmatrix} \mu_\pi \\ A_{n_1}^* \\ \vdots \\ A_{n_H}^* \end{bmatrix} + \begin{bmatrix} \iota' \\ B_{n-1}^{*\prime} \\ \vdots \\ B_{n_H}^{*\prime} \end{bmatrix} z_t + \begin{bmatrix} 0 \\ \omega_{n_1,t} \\ \vdots \\ \omega_{n_H,t} \end{bmatrix}. \tag{11}$$

$y_t^{n_h,N}$ is the daily nominal yield with an n_h-day maturity. With respect to Piazzesi (2010), a month of 21 trading days is assumed. So, n_h is 21h and 252h days for h-month and h-year maturities respectively. $\omega_{n_h,t}$ is the measurement error due to, for example, bid-ask spreads and zero-curve interpolation. Inflation in eq. (11) ensures its dynamic is consistent with the determining factors of real and nominal yields.[2]

3.2 A Linear Projection of Latent Variables

Khanthavit (2014a, 2014b) proposes an approach to estimate the model on a daily basis even though inflation is reported monthly. Latent factors z_t can be projected linearly by a set of η observed information variables $q_t' = [q_{0,t} = 1, q_{1,t}, \dots, q_{\eta-1},t]$. The projection equation is given by

$$z_t = b'q_t + v_t, \tag{12}$$

where $b' = \begin{bmatrix} b_{1,0}, b_{1,1}, \dots, b_{1,\eta-1} \\ \vdots \\ b_{K,0}, b_{K,1}, \dots, b_{K,\eta-1} \end{bmatrix}$ is the matrix of projection coefficients and $v_t' = [v_{1,t}, \dots, v_{K,t}]$ are projection errors. The linear projection approach follows Mishkin (1981) who estimated unobserved real yields by information variables. When $b'q_t + v_t$ is substituted for z_t in eq. (11), eq. (13.1) is obtained.

$$\begin{bmatrix} [\pi_t] \\ \begin{bmatrix} -n_1 y_t^{n_1,N} \\ \vdots \\ -n_H y_t^{n_H,N} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} [\mu_\pi] \\ \begin{bmatrix} A^*_{n_1} \\ \vdots \\ A^*_{n_H} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} [\iota'] \\ \begin{bmatrix} B^{*\prime}_{n_1} \\ \vdots \\ B^{*\prime}_{n_H} \end{bmatrix} \end{bmatrix} b'q_t + \begin{bmatrix} [v_{1,t}] \\ \begin{bmatrix} \omega_{n_1,t} + B^{*\prime}_{n_1} v_t \\ \vdots \\ \omega_{n_H,t} + B^{*\prime}_{n_H} v_t \end{bmatrix} \end{bmatrix} \tag{13.1}$$

$$= \begin{bmatrix} [b_{1,0} + \mu_\pi & b_{1,1} & \cdots & b_{1,\eta-1}] \\ \begin{bmatrix} A^*_{n_1} + B^{*\prime}_{n_1} b_0' & B^{*\prime}_{n_1} b_1' & \cdots & B^{*\prime}_{n_1} b_K' \\ \vdots & \vdots & & \vdots \\ A^*_{n_H} + B^{*\prime}_{n_H} b_0' & B^{*\prime}_{n_H} b_1' & \cdots & B^{*\prime}_{n_H} b_K' \end{bmatrix} \end{bmatrix} q_t + \begin{bmatrix} [v_{1,t}] \\ \begin{bmatrix} \omega_{n_1,t} + B^{*\prime}_{n_1} v_t \\ \vdots \\ \omega_{n_H,t} + B^{*\prime}_{n_H} v_t \end{bmatrix} \end{bmatrix} \tag{13.2}$$

$$= \alpha^T q_t + u_t. \tag{13.3}$$

[2] It is assumed factor $z_{1,t}$ correlates perfectly with inflation in order to simplify the model's structure. The first factor then can be interpreted as being inflation factor. The perfect correlation assumption is not restrictive. The factors are latent. When the first factor is inflation, the remaining factors can be rotated so that the fit of the model remains unchanged.

b'_{q-1} is column q of coefficient matrix b. Eq. (13.2) rearranges the coefficient vectors and matrices in eq. (13.1) by noticing that $q_{0,t} = 1$. $u_t = \begin{bmatrix} [v_{1,t}] \\ \begin{bmatrix} \omega_{n_1,t} + B^{*\prime}_{n_1} v_t \\ \vdots \\ \omega_{n_H,t} + B^{*\prime}_{n_H} v_t \end{bmatrix} \end{bmatrix}$ and $\alpha' =$

$$\begin{bmatrix} [b_{1,0} + \mu_\pi & b_{1,1} & \cdots & b_{1,\eta-1}] \\ \begin{bmatrix} A^*_{n_1} + B^{*\prime}_{n_1} b'_0 & B^{*\prime}_{n_1} b'_1 & \cdots & B^{*\prime}_{n_1} b'_K \\ \vdots & \vdots & & \vdots \\ A^*_{n_H} + B^{*\prime}_{n_H} b'_0 & B^{*\prime}_{n_H} b'_1 & \cdots & B^{*\prime}_{n_H} b'_K \end{bmatrix} \end{bmatrix}$$

so that eq. (13.3) is in a familiar regression format. The regression is linear in information variables q_t. But it is highly nonlinear in the parameters. Eq. (13.3) is important. All the regressors and regressants are observed. Now, the econometrician can use simple regressions for the estimation.

3.3 Identification of Necessary Parameters

3.3.1 The Parameters for Inflation

Similar to Hamilton and Yu (2012), Khanthavit (2014b) acknowledges that all the model parameters need not be estimated jointly but sequentially in steps. The projection coefficients $[b_{1,0}, b_{1,1} ..., b_{1,Q-1}]$ and the expectation μ_π for daily inflation can be inferred from monthly inflation data. Once these parameters are obtained, they can be employed together with daily nominal yield data to identify the remaining parameters. The two-step procedure offers an improvement because estimation errors in the $[b_{1,0}, b_{1,1} ..., b_{1,Q-1}]$ and μ_π estimates from the first step are lessened. Moreover, the use of daily nominal yields in the second step to estimate the remaining parameters should capture their daily motion better. Although its daily φ_{11} estimate from the two-step procedure is much larger than that in Khanthavit (2014a), Khanthavit (2014b) points out that it is still too small to be consistent with the AR(1) coefficient of monthly inflation.

This study notices further that the daily φ_{11} and σ_1^2 parameters for the inflation factor need not be estimated jointly with the remaining parameters in the second step but can be done by using monthly inflation data as those parameters in the first step. In order to do so, let $z_{1,t+1}$ in eq. (6) be the demeaned, daily inflation factor that follows an AR(1) process. $z_{1,t+1} = \varphi_{11} z_{1,t} + \varepsilon_{1,t+1}$, where σ_1^2 is the variance of ε_{t+1}. Furthermore, let $Z_{1,T+1}$ be the demeaned, monthly inflation factor. $Z_{1,T+1}$ is the sum of the daily inflation factor in month T + 1 and also follows an AR(1) process. $Z_{1,T+1} = \varphi^*_{11} Z_{1,T} + \xi_{1,T+1}$. The error term $\xi_{1,T+1}$ has a σ_ξ^2 variance. It can be shown that in a month of 21 days,

$$\varphi^*_{11} = (\varphi_{11})^{21} \tag{14.1}$$

$$\sigma^2_{\xi} = \sigma^2_1 \sum_{i=1}^{21-1} \left(\sum_{i=1}^{Min(2x21-i,i)} (\varphi_{11})^k \right)^2 \tag{14.2}$$

where $k = \begin{cases} Min(2x21 - i,i) - j & \text{if } j \leq 21 \\ (i + 21 - 1) + j & \text{if } j > 21 \end{cases}$. This functional relationship between monthly $[\varphi^*_{11}, \sigma^2_{\xi}]$ and daily $[\varphi_{11}, \sigma^2_1]$ enables the study to estimate daily $[\varphi_{11}, \sigma^2_1]$ for inflation directly from monthly inflation data and ensures the sizes of daily $[\varphi_{11}, \sigma^2_1]$ and monthly $[\varphi^*_{11},\sigma^2_{\xi}]$ are consistent. The study will estimate $[\varphi_{11}, \sigma^2_1]$ by maximum likelihood.

3.3.2 The Remaining Parameters

At this point, the remaining parameters that must be estimated are $\theta = [r, \lambda_1, ..., \lambda_K, \beta_{11}, \beta_{12}, ..., \beta_{KK}, \sigma^2_2, ..., \sigma^2_K, b_{2,0}, b_{2,1}, ..., b_{K,\eta-1}]$. One way to proceed in the second step is to follow Khanthavit (2014b) to estimate them by nonlinear SURE using daily nominal yield curves and the estimates from the first step. But as Hamilton and Wu (2012) pointed out and Khanthavit (2014b) himself experienced, the estimation is a numerical challenge because the objective surfaces are highly nonlinear and they behave badly. In order to lessen the computation time, this study applies Rothenberg's (1973) minimum-chi-square estimation which has been used successfully for example by Hamilton and Wu (2012) in similar problems.

Consider the following system of linear regressions of daily nominal yields on the information variables.

$$\begin{bmatrix} -n_1 y_t^{n_1,N} \\ \vdots \\ -n_H y_t^{n_H,N} \end{bmatrix} = \begin{bmatrix} C_{n_1,0}, C_{n_1,1}, ..., C_{n_1,\eta-1} \\ \vdots \\ C_{n_H,0}, C_{n_H,1}, ..., C_{n_H,\eta-1} \end{bmatrix} q_t + \begin{bmatrix} W_{n_1,t} \\ \vdots \\ W_{n_H,t} \end{bmatrix}, \tag{15}$$

where $\begin{bmatrix} C_{n_1,0}, C_{n_1,1}, ..., C_{n_1,\eta-1} \\ \vdots \\ C_{n_H,0}, C_{n_H,1}, ..., C_{n_H,\eta-1} \end{bmatrix}$ is the matrix of regression coefficients whose covariance matrix is R and $\begin{bmatrix} W_{n_1,t} \\ \vdots \\ W_{n_H,t} \end{bmatrix}$ is the vector of regression errors. Comparing eqs. (13.2) and (15) gives

$$\begin{bmatrix} C_{n_1,0}, C_{n_1,1}, ..., C_{n_1,\eta-1} \\ \vdots \\ C_{n_H,0}, C_{n_H,1}, ..., C_{n_H,\eta-1} \end{bmatrix} = \begin{bmatrix} A^*_{n_1} + B^{*\prime}_{n_1} b'_0 & B^{*\prime}_{n_1} b'_1 & \cdots & B^{*\prime}_{n_1} b'_K \\ & \vdots & & \\ A^*_{n_H} + B^{*\prime}_{n_H} b'_0 & B^{*\prime}_{n_H} b'_1 & \cdots & B^{*\prime}_{n_H} b'_K \end{bmatrix} \tag{16}$$

Define $C = Vech\left(\begin{bmatrix} C_{n_1,0}, C_{n_1,1}, \ldots, C_{n_1,\eta-1} \\ \vdots \\ C_{n_H,0}, C_{n_H,1}, \ldots, C_{n_H,\eta-1} \end{bmatrix}\right)$ as the vector of regression coefficients. $g(\theta) = Vech\left(\begin{bmatrix} A^*_{n_1} + B^{*\prime}_{n_1} b'_0 & B^{*\prime}_{n_1} b'_1 & \cdots & B^{*\prime}_{n_1} b'_K \\ & \vdots & & \\ A^*_{n_H} + B^{*\prime}_{n_H} b'_0 & B^{*\prime}_{n_H} b'_1 & \cdots & B^{*\prime}_{n_H} b'_K \end{bmatrix}\right)$ is the vector of functions $g(\theta)$ of the remaining parameters θ that describe the shape of nominal curves. Rothenberg (1973) shows that the remaining parameters can be identified by minimizing the chi-square statistic χ^2 in eq. (17) with respect to θ.

$$\chi^2 = \tau\,[C - g(\theta)]' R^{-1}\,[C - g(\theta)]. \tag{17}$$

where τ is the number of observations. The minimizers $\hat{\theta}$ have the property $\sqrt{\tau}\,(\hat{\theta} - \theta) \rightarrow Normal\left(0, \left[\left(\frac{\partial g(\theta)}{\partial \theta'}\right)' R^{-1} \left(\frac{\partial g(\theta)}{\partial \theta'}\right)\right]^{-1}\right)$. The study estimates coefficient vector C and its covariance matrix R by seemingly unrelated regression estimation (SURE) because the technique does not require a normality assumption.

4. The Data

4.1 Samples and Data Sources

The study applies the minimum-chi-square technique to estimate daily real yields of up to 15-year maturity in Thailand’s bond market. The sample period is from July 2, 2001 to May 30, 2014. It starts on the first day the Thai Bond Market Association reported nominal yield curves of up to 15 years. The use of up-to-15-year curves is important because the resulting real yields are more consistent with the durations of inflation-linked bonds currently traded on the market. The nominal yield data are daily for 1-month, 3-month, 6-month and 1-year up to 15-year maturities, with one-year increments, from the Thai Bond Market Association. The inflation is logged monthly inflation, computed using the headline consumer price index from the Bureau of Trade and Economic Indices, Ministry of Commerce.

Table 1 reports the descriptive statistics of inflation and nominal yields. The average inflation is 2.7205%. This estimate will serve as the expected inflation in the estimation of the remaining parameters. The term structure of average nominal yields has a normal shape, while the volatility structure has a “U” shape. The statistics are slightly different from those reported by Khanthavit (2014a, 2014b). These differences are expected due to their different sample periods and sample maturities. In the last column of Table 1, the study tests and rejects the normality assumptions for the inflation and nominal yield data. The Jarque-Bera (JB) statistics are very large and their p values are zero. The rejection supports the use of SURE to estimate the coefficient vector C to be performed below.

Table 1

Descriptive Statistics

Variables	Average	Max	Min	Std.	Skew.	E. Kurt.	JB Stat.
Inflation	2.7205%	25.8264%	-36.7878%	6.5453%	-1.3341	9.9468	648.9666***
1M	2.4304%	5.0333%	0.7799%	1.0712%	0.6090	-0.2138	195.0029***
3M	2.4976%	5.0536%	0.7981%	1.0570%	0.5965	-0.1987	186.4879***
6M	2.5939%	5.2136%	0.8633%	1.0465%	0.5673	-0.2588	172.6802***
1Y	2.7127%	5.3154%	0.9314%	1.0430%	0.5615	-0.3167	173.5665***
2Y	2.9915%	5.5432%	1.1781%	1.0356%	0.6275	-0.1994	205.8809***
3Y	3.2236%	5.8372%	1.3491%	0.9901%	0.6214	-0.0554	197.3526***
4Y	3.4728%	6.1637%	1.4515%	0.9264%	0.5366	0.1238	148.7831***
5Y	3.7005%	6.3980%	1.5680%	0.9082%	0.4785	0.0596	117.2136***
6Y	3.9172%	6.6710%	1.7383%	0.8791%	0.4130	-0.0376	87.1869***
7Y	4.1201%	6.7853%	1.8978%	0.8516%	0.3742	-0.0978	72.6456***
8Y	4.2629%	6.8614%	2.0604%	0.8713%	0.3874	-0.3481	91.9947***
9Y	4.3705%	6.9546%	2.2364%	0.8993%	0.4246	-0.3796	110.3067***
10Y	4.5080%	7.1884%	2.4839%	0.9271%	0.4590	-0.4301	131.0385***
11Y	4.6681%	7.2306%	2.7071%	0.9437%	0.4448	-0.5850	144.5074***
12Y	4.7596%	7.2657%	2.8725%	0.9349%	0.4875	-0.4874	151.4829***
13Y	4.8278%	7.3534%	2.9819%	0.9347%	0.5081	-0.4632	159.0171***
14Y	4.8894%	7.5426%	3.1332%	0.9484%	0.5018	-0.4706	156.6484***
15Y	4.9581%	7.6510%	3.2018%	0.9568%	0.5153	-0.4462	160.7953***

Note: The statistics for inflation is monthly, while those for nominal yields are daily. *** = Significance at a 99% confidence level.

4.2 Information Variables

$\eta = 5$ information variables are considered in the projection of the inflation factor in eq. (13.2) and in the regression of nominal yields in eq. (15). The first variable is a constant. The remainders are 1-day lagged Bjork-Christensen (1999) beta shape factors. As Khanthavit (2013) reported, these factors could predict Thailand's nominal term structure accurately.

To check for projection ability, the study regresses daily nominal yields on daily information variables and regresses monthly inflation on monthly-aggregate information variables. From eqs. (13.2) and (15) if the information variables are able project the latent factors, the regression coefficients must be significant. The results are in Table 2. The coefficients for the nominal yields are highly significant. For inflation, the coefficients for beta shape factors 2 and 4 are significant at a 95% confidence level. Based on these results, the study concludes that the chosen information variables have projection ability.

It is noted that the R^2's for nominal yields are very high. All are over 99%. The high R^2's and highly significant coefficients can be explained by Khanthavit's (2013) observation that the nominal yields and beta shape factors were long-memory, near-I(1) variables. So, the results were similar to the ones from co-integration regressions.

Table 2

Tests for Projection Ability of Information Variables

Variables	Constant	Beta F. 1	Beta F. 2	Beta F. 3	Beta F. 4	R^2
Inflation	0.0001	-0.0044	-0.0050**	0.0022	-0.0069**	0.0307
1M	0.0004***	0.9968***	0.9805***	0.0308***	0.9468***	0.9982
3M	0.0000	0.9980***	0.9244***	0.0702***	0.8586***	0.9990
6M	-0.0002***	0.9998***	0.8500***	0.1227***	0.7443***	0.9976
1Y	-0.0008***	1.0096***	0.7655***	0.1825***	0.6164***	0.9969
2Y	0.0001	1.0009***	0.5948***	0.2825***	0.3848***	0.9969
3Y	0.0004***	0.9904***	0.4689***	0.3140***	0.2531***	0.9956
4Y	0.0017***	0.9667***	0.3748***	0.3019***	0.1887***	0.9931
5Y	0.0000	1.0093***	0.3359***	0.2773***	0.1791***	0.9925
6Y	0.0000	1.0039***	0.2792***	0.2521***	0.1434***	0.9926
7Y	0.0007***	0.9830***	0.2371***	0.2142***	0.1292***	0.9888
8Y	-0.0007***	1.0082***	0.2240***	0.1867***	0.1328***	0.9902
9Y	-0.0014***	1.0074***	0.1475***	0.1725***	0.0634***	0.9894
10Y	-0.0014***	1.0074***	0.1475***	0.1725***	0.0634***	0.9903
11Y	-0.0008***	0.9978***	0.1315***	0.1443***	0.0641***	0.9925
12Y	0.0002***	0.9945***	0.1440***	0.1323***	0.0807***	0.9952
13Y	0.0008***	0.9887***	0.1401***	0.1328***	0.0707***	0.9955
14Y	0.0010***	0.9893***	0.1281***	0.1376***	0.0514***	0.9932
15Y	0.0014***	0.9887***	0.1212***	0.1361***	0.0454***	0.9920

Note: ** and *** = Significance at 95% and 99% confidence levels, respectively. The statistics for the inflation are estimated using monthly inflation and the sum of information variables in the month, while those of nominal yields are estimated using daily yields and information variables.

5. Empirical Results

5.1 The Number of Factors

Khanthavit (2014a, 2014b) reports that the first two principal components can explain 97.92% of the variation of Thailand's nominal yields, hence imposing a two-factor interest model. Because this study considers a different sample period and longer yield curves of up to 15-year maturity, a principal component analysis is conducted to reexamine the number of factors. The results are in Table 3.

Table 3

Principal Component Analysis

Principal Component	Contribution	Accumulated Contribution
1	75.1770%	75.1770%
2	22.2830%	97.4600%
3	1.8931%	99.3531%
4 and Beyond	0.6469%	100.0000%

From the table, the first two factors can explain 97.46% of the yield's variation. This finding is very close to those in Khanthavit (2014a, and 2014b). It leads the study to impose a two-factor model for the following analyses.

5.2 Parameter Estimates

The parameter estimates are reported in Table 4. The estimate μ_π is a scaled average of monthly inflation data, while the projection coefficients $[b_{1,0}, b_{1,1}, ..., b_{1,4}]$ for the inflation factor are from the regression of demeaned monthly inflation on the monthly aggregate information variables. The statistics φ_{11} and σ_1 are estimated by maximum likelihood, using demeaned monthly inflation data together with the relationships in eq. (14). The remaining parameters are from minimum-chi-square estimation.

An important motivation of this study is numerical challenging of parameter estimation in Khanthavit (2014a, 2014b). If the proposed minimum-chi-square estimation is successful, convergence must be fast and computation time must be decreasing substantially. It is found that convergence is obtained very fast in about fewer than 200 iterations. The computation times falls from almost 5 hours in the previous studies to about less than 10 minutes. However, the solutions are mostly local. This finding differs from Hamilton and Wu (2012) who applied minimum-chi-square estimation in similar empirical problems but found almost the same solutions regardless of the starting values. To ensure that better solution is obtained and closer to the global optimum, the study chooses 50 sets of starting values at random. The minimum-chi-square estimates in Table 4 are the ones that yield the minimum chi-square objective value.

Table 4

Parameter Estimates

Parameters	Value
$\bar{r} \times 25200$	1.2691***
γ_1	-1.6462***
γ_2	-2.2430E-04
λ_1	48.3680***
λ_2	-4.0573
β_{11}	-279682.6900***
β_{12}	-189.7953
β_{21}	-38569.2540
β_{22}	-16208.3120
φ_{11}	0.7953***
φ_{21}	0.1901
φ_{22}	0.9960***
σ_1	5.4759E-04***
σ_2	4.3327E-04
$\mu_\pi \times 25200$	2.7209***
$b_{1,0}$	0.0001
$b_{1,1}$	-0.0044
$b_{1,2}$	-0.0050**
$b_{1,3}$	0.0022
$b_{1,4}$	-0.0069**
$b_{2,0}$	0.4253
$b_{2,1}$	-7.2333
$b_{2,2}$	-6.4590
$b_{2,3}$	-1.6255
$b_{2,4}$	-5.0511

Note: **, and *** = Significance at 95% and 99% confidence levels, respectively. μ_π is a monthly average divided by 21. $b_{1,0}$, ..., $b_{1,4}$ are the regression coefficients of the monthly inflation on the sum of the information variables in the month. σ_1 and φ_{11} are estimated separately from μ_π and monthly inflation data by maximum likelihood. The remaining estimates are from minimum-chi-square estimation.

Turn next to the resulting estimates. The estimates are not very close with the ones reported in Khanthavit (2014a, 2014b). These differences are expected due to at least three reasons. Firstly, our sample periods and nominal yield curves differ. Secondly, this study estimates φ_{11} and σ_1 directly from the inflation data in addition to the expected inflation μ_π and the projection coefficients $[b_{1,0}\ b_{1,1}, ..., b_{1,4}]$ in Khanthavit (2014b). Thirdly, the values of the estimates in the second step partly depend on those different values in the first step.

It is important to note that the direct estimation of φ_{11} from inflation data offers improvement. The estimate equals 0.7953, it is much larger than (0.0179, 0.5514) reported by Khanthavit (2014a, 2014b), and it lies much closer to 0.9496—the daily AR(1) coefficient implied from the 0.3376 monthly value.

5.2 Specification Tests

I follow Ang et al. (2008) to conduct specification tests for the model. If the model fits, the moments of sample and fitted nominal yields should not differ. Comparison of the means, standard deviations, skewnesses and excess kurtoses are in Table 5. The numbers in the first lines are for fitted yields and those in the second lines are their deviations from the sample moments. Significance is based on the White (2000) procedure.

The deviations are small and not significant for all the moments and across maturities, except for the standard deviations of 3-year and longer yields. The significance of standard deviations was also reported for most specifications of the Ang et al. (2008) model. With respect to the small number of significant cases and when compared and contrast with the ones reported by the previous study, I conclude that the model satisfactorily fit Thailand's nominal yields. However, it performs slightly less well than do Khanthavit (2014a, 2014b) in certain aspects such as larger mean deviations. The poorer performance should be expected. While this study and Khanthavit (2014a, 2014b) consider the same two factor model, the same two factors in this study must fit much longer maturities and many more data points.

Table 5
Specification Tests

Maturity	Descriptive Statistics			
	Mean	Std.	Skew.	E. Kurt
1M	2.4762	1.0462	0.6234	-0.1187
	0.0458	-0.0250	0.0144	0.0951
3M	2.4660	1.1243	0.5034	-0.3973
	-0.0317	0.0673	-0.0930	-0.1986
6M	2.5427	1.1280	0.4612	-0.4673
	-0.0512	0.0815	-0.1061	-0.2085
1Y	2.7191	1.0865	0.4399	-0.4995
	0.0064	0.0436	-0.1216	-0.1828
2Y	3.0509	0.9877	0.4293	-0.5147
	0.0594	-0.0478	-0.1982	-0.3153
3Y	3.3422	0.8967	0.4259	-0.5196
	0.1186	-0.0934*	-0.1956	-0.4641
4Y	3.5967	0.8161	0.4242	-0.5219
	0.1239	-0.1103**	-0.1124	-0.6458
5Y	3.8194	0.7452	0.4232	-0.5233
	0.1189	-0.1630***	-0.0553	-0.5829
6Y	4.0148	0.6829	0.4226	-0.5242
	0.0976	-0.1962***	0.0095	-0.4866
7Y	4.1866	0.6279	0.4222	-0.5248
	0.0665	-0.2236***	0.0479	-0.4269
8Y	4.3382	0.5794	0.4218	-0.5252
	0.0753	-0.2919***	0.0344	-0.1771
9Y	4.4723	0.5365	0.4216	-0.5255
	0.1019	-0.3628***	-0.0030	-0.1459
10Y	4.5914	0.4983	0.4215	-0.5257
	0.0834	-0.4288***	-0.0376	-0.0956
11Y	4.6973	0.4643	0.4213	-0.5259
	0.0293	-0.4794***	-0.0234	0.0590
12Y	4.7920	0.4340	0.4212	-0.5261
	0.0324	-0.5009***	-0.0662	-0.0386
13Y	4.8767	0.4068	0.4212	-0.5262
	0.0489	-0.5279***	-0.0869	-0.0629
14Y	4.9529	0.3823	0.4211	-0.5262
	0.0634	-0.5660***	-0.0807	-0.0557
15Y	5.0215	0.3603	0.4211	-0.5263
	0.0633	-0.5965***	-0.0942	-0.0801

Note: *, ** and *** = Significance at 90%, 95% and 99% confidence levels, respectively. The statistics on the upper lines are those of the fitted yields and the ones on the lower lines are the deviations from sample statistics.

5.3 The Resulting Daily Real Yields

The estimation of daily real yields is successful and very fast. In Table 6, the term structure of Thailand's real yields is time varying. Its average has a normal shape. The averages for 1-month up to 1-year maturities are negative but rising. They turn positive for a 2-year maturity and over. When compared to those in Khanthavit (2014a), the curves are steeper. The averages are about 50 basis points lower for short rates and about 100 basis points higher for long rates. And when compared with those in Khanthavit (2014b), the curves look similar. The averages are about the same for short rates but are about 40 basis points higher for long rates.

Khanthavit (2014b) argues that its estimates of real curves are more accurate than those in Khanthavit (2014a). With respect to the tests in Table 5, the study accepts that its performance is poorer than Khanthavit's (2014b) performance. Yet their average real curves are close and similar. Because the curves in this study cover longer maturities of up to 15 years and the computation consumes significantly less time, for Thailand it is more practical to estimate and use these curves.

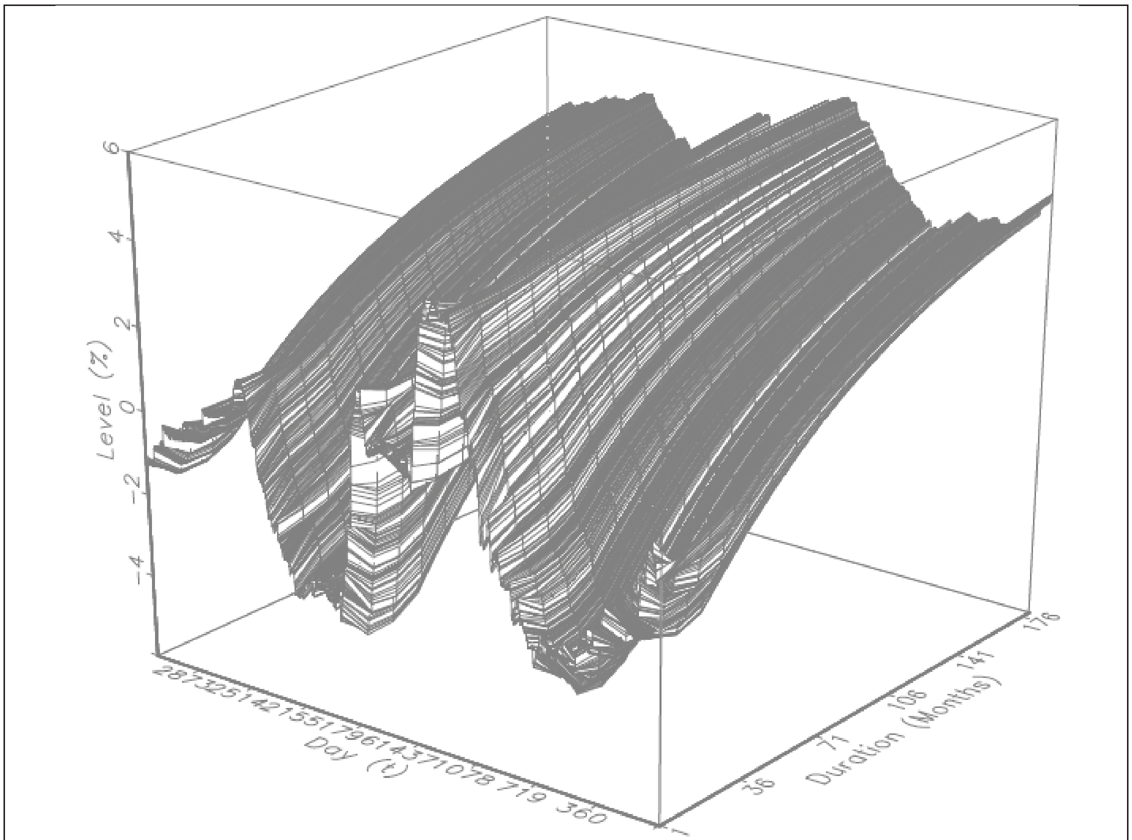
6. Conclusion

In previous studies, Khanthavit (2014a, 2014b) proposed a linear projection approach to estimate real yields on a daily basis. The proposed approach is important because the daily results promote active trading of the securities and closer monitoring of the economy. Nevertheless the computation is very slow, therefore not very practical. This study applies a fast minimum-chi-square estimation technique to enhance the speed. The study finds a normal shape for the average daily real curve. The average 1-month real yield is -0.5446%. The yields are rising with maturities. The average 10- and 15-year yields are 3.1215% and 3.9199%. Although they are less precise than those in previous studies because the same two factors must fit longer curves and more data points, the curves should be more practical and useful for Thailand's financial market. After all, the curves cover up to 15-year maturity and they can be estimated using less than one thirtieth of the computation time.

The study also estimates certain components, e.g. inflation premiums, real yield premiums and expected inflations, of and their percentage contributions to nominal yields. To save space, they are not reported here but interested readers can obtain these statistics from the author.

Table 6

Daily Real Yields



Maturity	Average	Max	Min	Std.
1M	-0.5446***	4.3961	-3.6223	1.9526
3M	-0.7349***	4.2726	-4.2286	2.1106
6M	-0.6412***	4.3802	-4.2055	2.1253
1Y	-0.3375***	4.4965	-3.8349	2.0514
2Y	0.2674***	4.6595	-2.9521	1.8668
3Y	0.8050***	4.7915	-2.1294	1.6953
4Y	1.2763***	4.9042	-1.3997	1.5432
5Y	1.6891***	5.0018	-0.7572	1.4093
6Y	2.0515***	5.0868	-0.1917	1.2915
7Y	2.3703***	5.1614	0.3066	1.1876
8Y	2.6516***	5.2270	0.7467	1.0959
9Y	2.9005***	5.2849	1.1364	1.0147
10Y	3.1215***	5.3362	1.4825	0.9425
11Y	3.3182***	5.3819	1.7907	0.8782
12Y	3.4938***	5.4226	2.0661	0.8208
13Y	3.6511***	5.4591	2.3128	0.7694
14Y	3.7925***	5.4918	2.5344	0.7232
15Y	3.9199***	5.5213	2.7342	0.6815

Note: *** = Significance at a 99% confidence level. Day (t=1) is July 2, 2001 and Day (t=3161) is May 30, 2014

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