Degree of Operating Leverage (DOL), Operating Profit Margin and Growth

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Abstract

This paper has three objectives. Firstly, we explore relationships between growth rates of sales, cost, and profit and their associated margins. We provide formulae that relate all these variables and derive the implications. Secondly, we show further that if we assume a constant fixed cost and a constant variable cost ratio, then we can deduce contribution margin ratio (cm), variable cost as a proportion of sales (vc) and fixed cost as a proportion of sales (fc) from operating margin ratio (om) and degree of operating leverage (DOL). In addition, we can also infer sales breakeven simply from sales and profit growth rates.

Thirdly, we relax the above assumptions by assuming a known change in fixed cost and variable cost ratio. These changes could be approximated from financial statements and industry price index series. Our approach could still estimate cm, vc, fc and om. In addition, we also extend Arellano (2007)'s method to show a relationship between a change in margin and fixed cost in this case.

We also provide implications in terms of financial statement analysis. One of the key implications is that if sales growth is higher than cost growth, then a firm with a higher operating profit margin ratio will have a lower profit growth compared to that of another firm with a lower margin. The result is reversed when sales growth is lower than cost growth.

Keyword: DOL, degree of operating leverage, profit margin, growth, fixed cost, variable cost, contribution margin

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ระดับความเสี่ยงในการดำเนินงาน (Degree of Operating Leverage: DOL), อัตราส่วนกำไรจากการดำเนินงาน (Operating Profit Margin) และอัตราการเติบโต (Growth)

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บทคัดย่อ

บทความนี้ศึกษาถึง 1) ความสัมพันธ์ระหว่างอัตราการโตของยอดขาย ต้นทุน กำไร และอัตราส่วนกำไร (Margin) โดยเสนอสมการซึ่งเชื่อมระหว่างตัวแปรเหล่านี้ 2) วิธีในการประมาณค่า อัตราส่วนต้นทุนคงที่ อัตราส่วนต้นทุนผันแปร อัตราส่วนกำไรที่มีเหนือกว่าต้นทุนผันแปร (Contribution Margin) จากค่าอัตราส่วนกำไรจากการดำเนินงาน (Operating Profit Margin Ratio) และระดับความเสี่ยงในการดำเนินงาน (Degree of Operating Leverage: DOL) และหาค่ายอดขายคุ้มทุน (Sales Breakeven) จากอัตราการเติบโตของยอดขายและกำไร ภายใต้สมมุติฐานที่ว่า ต้นทุนคงที่ไม่เปลี่ยนแปลง และอัตราส่วนต้นทุนผันแปรไม่เปลี่ยนแปลง 3) วิธีในการประมาณค่าตัวแปรข้างต้น โดยการผ่อนคลายสมมุติฐานให้ต้นทุนคงที่และอัตราส่วนต้นทุนผันแปรเปลี่ยนแปลงได้ โดยการเปลี่ยนแปลงดังกล่าวสามารถประมาณได้จากรายการในงบการเงินและจากดัชนีราคาของแต่ละอุตสาหกรรม นอกจากนั้นงานวิจัยนี้ยังได้เพิ่มเติมวิธีการของ Arellano (2007) ซึ่งแสดงความสัมพันธ์ระหว่างการเปลี่ยนแปลงของอัตราส่วนกำไรและต้นทุนคงที่

ผลการศึกษาได้แสดงให้เห็นว่าถ้าอัตราการเติบโตของยอดขายมีค่ามากกว่าอัตราการเติบโตของต้นทุน กำไรของบริษัทซึ่งมีอัตราส่วนกำไรสูงกว่าจะเติบโตในอัตราที่น้อยกว่ากำไรของบริษัทที่มีอัตราส่วนกำไรต่ำกว่า โดยผลจะเป็นตรงกันข้ามถ้าอัตราการเติบโตของยอดขายมีค่าน้อยกว่าอัตราการเติบโตของต้นทุน

คำสำคัญ: DOL, Degree of Operating Leverage, อัตราส่วนกำไร, การเติบโต, ต้นทุนคงที่, ต้นทุนผันแปร, กำไรส่วนเกิน

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1. Introduction

Most business students study financial statement analysis and cost-volume-profit (CVP) analysis in their accounting classes where they are exposed to trend analysis, common size analysis, percentage change analysis (Koh, Ang, Brigham, & Ehrhardt, 2014), degree of operating leverage (DOL) and break-even analysis (Maher, Stickney, & Weil, 2011). They also learn about margins whether in the forms of gross profit margin, operating margin or net income margin. However, relationships among growth rates of sales and costs and margins are not shown nor derived explicitly. In addition, a standard management accounting class would cover the concept of contribution margin but provide no link to reported numbers in financial statements.

This paper addresses the above limitations by bridging financial accounting and management accounting. We propose novel analytical techniques that can be used in financial statement analysis. Firstly, we provide formulae that relate growth rates of sales, cost and profit and its associated margins. If we know any three of these variables, we can then deduce the rest. These formulae are particularly useful when analyzing numbers from financial highlight statistics, which do not reveal full information. Our results show that if sales growth is higher than cost growth, then a firm with a higher operating margin ratio (om) will have a lower profit growth compared to that of another firm with a lower margin. The result is reversed when sales growth is lower than cost growth.

Secondly, we show a method to deduce contribution margin ratio (cm), variable cost as a proportion of sales (vc) and fixed cost as a proportion of sales (fc) from reported financial numbers if we are willing to assume a constant fixed cost and a constant variable cost ratio. In addition, we can also infer sales breakeven simply from sales and profit growth rates.

Thirdly, we extend the above approach by assuming a known change in fixed cost and variable cost ratio. These changes could be approximated from financial statements and industry price index series. Our approach could still estimate cm, vc, fc and om. We also extend Arellano (2007)'s method to shows a relationship between a change in margin and fixed cost in this case.

The organization of this paper is as follows. Section 1 is this introduction. Section 2 provides literature review. Section 3 derives relationships among DOL, operating margin ratio (om) and growth, provides numerical examples, and illustrates applications of the technique. Finally, section 4 discusses implications and concludes.

2. Literature Review

The degree of operating leverage is basically an elasticity of operating profit, measured by EBIT (Earnings Before Interest and Taxes), with respect to sales (Stelk, Park, & Dugan, 2015). For example, if DOL is two, it simply means that if sales were to increase by one percentage point then the operating profit would increase by two-percentage point. Zivney (2000) provides the formulae for degree of operating leverage (DOL). The formulae could be stated in the form of quantity sold (Q) or sales (S).

$$\begin{aligned} DOL &= \frac{\%\Delta EBIT}{\%\Delta Sales} = \frac{\left(\frac{\Delta EBIT}{EBIT}\right)}{\left(\frac{\Delta Sales}{Sales}\right)} \\ &= \frac{Q(P-V)}{Q(P-V)-FC} = \frac{S-VC}{S-VC-FC} = \frac{EBIT+FC}{EBIT} = \frac{S}{S-S_{BE}} \\ &= \frac{Contribution\ Margin\ Ratio}{Operating\ Profit\ Margin\ Ratio} = \frac{cm}{om} \end{aligned} \quad (1)$$

Note: EBIT = operating income, S = sales, S_{BE} = sales breakeven, Q = quantity sold, P = price per unit, V = variable cost per unit, FC = fixed cost, VC = variable cost, cm = contribution margin ratio, om = operating margin ratio

Aside form the formula approach[1] as stated above, there are two popular empirical methods to estimate DOL (Stelk et al., 2015). The first method by Mandelker and Rhee (1984) is based on the concept of elasticity. They use the time-series regression to estimate the elasticity between operating profits (EBIT) and sales. Mathematically, they use the following regression.

$$ln(EBIT_{it}) = a_i + c_i \times ln(Sales_{it}) + \mu_{it}$$

The symbol "i" and "t" stand for firm "i" and time "t", respectively. The variable "μ_{it}" is simply the error term. The coefficient "c_i" is the DOL of firm "i". Basically, it is the elasticity of operating profits with respect to sales.

O'Brien and Vanderheiden (1987) criticize the above formulation that it fails to control for growth in operating earnings and sales. They propose the new method based on the percentage deviation of operating profits and sales from their own trends. Mathematically, they estimate the percentage deviations from these regressions.

$$ln(EBIT_t) = ln(EBIT_0) + t \times g_{EBIT} + \mu_t^{EBIT}$$
$$ln(Sales_t) = ln(Sales_0) + t \times g_{Sales} + \mu_t^{Sales}$$

[1] This is also the approach of this paper.

Then, in their second stage, they apply the following regression to estimate DOL.

$$\mu_t^{EBIT} = DOL \times \mu_t^{Sales} + \varepsilon_t$$

They argue that this DOL measures the average sensitivity of the percentage deviation of the operating profits from its trend relative to that of sales.

Stelk et al. (2015) study whether which of the above method is more accurate in terms of estimating a firm's DOL. They find that the second method (O'Brien & Vanderheiden, 1987) is better, so they advocate its use in practical applications.

In terms of relationships between growth rates and fixed costs, Arellano (2007) proposes an equation that relates operating profit margin ratio, sales growth rate, and fixed cost as a proportion of sales. He correctly points out that the frequently used percent-of-sales method in a financial projection implicitly assumes that all costs are variable. This implies that operating profit margin ratio remains constant. This may be reasonable in the long run, but there are fixed costs in the short run. The existence of fixed cost will cause total cost as a proportion to sales to decline (rise) when sales increase (decrease). Consequently, operating profit margin ratio will increase (decrease) as sales increase (decrease) if we assume that the variable cost ratio is constant. The equation is shown below.

$$om_1 = om_0 + \left(\frac{s}{1+s}\right) \times fc \tag{2}$$

Note: om_1 = operating profit margin ratio in period 1, om_0 = operating profit margin ratio in period 0, s = sales growth, fc = fixed cost as a proportion of sales in period 0

3. Relationships Among DOL, Operating Profit Margin (PM) and Growth

This section is separated into three parts. Each part provides both theoretical models and numerical examples. The first part shows relationships among operating profit margin ratio, sales growth, cost growth, and profit growth. The second part discusses relationships among contribution margin ratio (cm), variable cost as a proportion of sales (vc), fixed cost as a proportion of sales (fc), operating margin ratio (om), degree of operating leverage (DOL) and sales breakeven (S_{BE}). In addition, we show a method to infer sales breakeven simply from sales and profit growth rates and to infer a fixed cost ratio simply from a change in operating profit margin ratio and sales growth (Arellano, 2007). Unfortunately, these relationships are based on two underpinning assumptions, namely, a constant total fixed cost and a constant variable cost ratio.

The third part relaxes the above assumptions and extends our approach to the case of a known change in total fixed cost and variable cost ratio. In application, these changes could be approximated from financial statements and industry price index series.

3.1 Relationships among operating profit margin ratio, sales growth, cost growth and profit growth

3.1.1 Theoretical models

We begin with the definition of profit. Let "NI" be profit in the form of earnings before interest and taxes (EBIT) or in other words, operating profit. We then define "S" as sales or total revenue in case that a firm has other incomes and define "C" as total operating cost.[2] The proportion of costs over sales is represented by the variable "w" (w = C/S). Obviously, profit is the difference between sales and costs.

We analyze two periods, namely period "0" and "1".

$$\begin{aligned} w_0 &= \frac{C_0}{S_0} \\ NI_0 &= S_0 - C_0 = S_0 - w_0.S_0 = (1 - w_0)S_0 \\ NI_1 &= S_1 - C_1 = (1 + s)S_0 - (1 + c)w_0.S_0 = [(1 + s) - (1 + c)w_0]S_0 \end{aligned} \tag{3}$$

The variable "s" and "c" stand for growth rate of sales and total costs, respectively. The variable "w_0" is the cost ratio or, in other words, a proportion of total costs over sales in period zero.

If we know the rates of growth of total costs and sales, we could recalculate the proportion of total costs over sales in period one. Similarly, we derive the relationship between initial margin (om_0) and current margin (om_1).

$$w_1 = \frac{C_1}{S_1} = \frac{C_0(1+c)}{S_0(1+s)} = \left(\frac{C_0}{S_0}\right)\frac{(1+c)}{(1+s)} = w_0\frac{(1+c)}{(1+s)} \tag{4}$$

$$om = 1 - c$$

$$om_1 = 1 - w_1 = 1 - w_0\frac{(1+c)}{(1+s)} = 1 - (1 - om_0)\frac{(1+c)}{(1+s)} \tag{5}$$

We define operating margin ratio (om) as one minus cost ratio (om = 1 – c). The profit growth rate (g) is derived in the following equation.

$$\begin{aligned} g &= \frac{(NI_1 - NI_0)}{NI_0} = \frac{NI_1}{NI_0} - 1 \\ &= \frac{[(1+s) - (1+c)w_0]}{(1 - w_0)} - 1 \\ g &= \frac{(s - w_0.c)}{(1 - w_0)} = \frac{s - (1 - om_0).c}{om_0} \end{aligned} \tag{6}$$

[2] The formulae, in fact, work even when we define "NI", "S" and "C" differently as long as "NI = S – C". For example, if we define "NI" as gross profit, then we must define "S" as sales of goods and "C" as costs of goods sold. Or, if we define "NI" as net income, then we must define "S" as total revenue and "C" as total cost including operating costs, finance costs and tax expenses.

We can also restate sales growth as a linear combination of profit growth and cost growth. The formula turns messier if we use current cost ratio (w_1) or current operating profit margin (om_1) than initial cost ratio (w_0) or initial operating profit margin (om_0).

$$s=(1-w_0).g+w_0.c=om_0.g+(1-om_0).c \tag{7}$$

$$s=\frac{(1-w_1).g+(g+w_1).c}{\left[1+w_1.g+(1-w_1).c\right]}=\frac{om_1.g+(1+g-om_1).c}{\left[1+(1-om_1).g+om_1.c\right]} \tag{8}$$

If we solve the profit growth (g) equation (Eq. 6) for the initial cost ratio (w_0), we then get the equation that links the initial cost ratio (w_0) or the current cost ratio (w_1) to profit growth (g) and cost growth (c).

$$w_0=\frac{(g-s)}{(g-c)} \tag{9}$$

$$w_1=w_0.\frac{(1+c)}{(1+s)}=\frac{(g-s)}{(g-c)}.\frac{(1+c)}{(1+s)} \tag{10}$$

If we substitute the current cost ratio (w_1) into the profit growth equation (Eq. 6), then we can find the relationship between profit growth (g), cost growth, and current cost ratio (w_1).

$$g=\frac{(s-w_0.c)}{(1-w_0)}=\frac{s-(1-om_0).c}{om_0}$$

$$g=\frac{\left[\frac{(s-w_1.c)}{(1-w_1)}\right]+(s.c)}{\left[1-\frac{(w_1.s-c)}{(1-w_1)}\right]}=\frac{\left[\frac{s-(1-om_1).c}{om_1}\right]+(s.c)}{\left[1-\frac{(1-om_1).s-c}{om_1}\right]} \tag{11}$$

We can also restate the profit growth equation (Eq. 6) to write cost growth (c) as a function of sales growth and profit growth.

$$c=g+\frac{1}{w_0}(s-g)=g+\frac{1}{(1-om_0)}(s-g) \tag{12}$$

$$c=\frac{\left(g+\frac{1}{w_1}\frac{(s-g)}{(1+s)}\right)}{\left(1-\frac{1}{w_1}\frac{(s-g)}{(1+s)}\right)}=\frac{\left(g+\frac{1}{(1-om_1)}\frac{(s-g)}{(1+s)}\right)}{\left(1-\frac{1}{(1-om_1)}\frac{(s-g)}{(1+s)}\right)} \tag{13}$$

The above equations show relationships between sales, cost and profit growth with operating profit margin ratio (om) either in the form of initial one or current one. If we know any three of these variables, we can then deduce the rest.

The profit growth equation (Eq. 6) is restated here to discuss its implication. The equation could be re-arranged to get the following two equations.

$$g = \frac{(s - w_0.c)}{(1 - w_0)} = \frac{s - (1 - om_0).c}{om_0} \tag{6}$$

$$g = s + \frac{w_0}{(1 - w_0)}.(s - c) = s + \frac{(1 - om_0)}{om_0}.(s - c) \tag{14}$$

$$g = c + \frac{1}{(1 - w_0)}.(s - c) = c + \frac{1}{om_0}.(s - c) \tag{15}$$

The key implication here is that if sales growth is higher than cost growth, then a firm with a higher (lower) initial operating profit margin ratio (om_0) will have a lower (higher) profit growth compared to that of another firm with a lower (higher) margin. Implicitly, we assume that the initial profit margins are positive. Of course, we obviously make this comparison under the condition that sales growth rates are the same for both firms and cost growth rates are also the same for both firms.

In contrast, if sales growth is lower than cost growth, then a firm with a higher (lower) initial operating profit margin ratio (om_0) will have a higher (lower) profit growth compared to that of another firm with a lower (higher) operating profit margin ratio.

This fact helps explain why turnaround firms tend to grow much faster than more mature firms during economic recovery, where sales growth tends to be higher than cost growth.

Even though the above equation and conclusion is mathematically correct, it is not quite intuitive. To get a better understanding of the reason, we can write down the following equation to show the percentage change in operating profit margin ratio.

$$\begin{aligned}
\Delta om = om_1 - om_0 &= \left(\frac{S_1 - C_1}{S_1}\right) - \left(\frac{S_0 - C_0}{S_0}\right) = \left(\frac{(1+s)S_0 - (1+c)C_0}{(1+s)S_0}\right) - \left(\frac{S_0 - C_0}{S_0}\right) \\
&= \left(1 - \frac{(1+c)C_0}{(1+s)S_0}\right) - \left(1 - \frac{C_0}{S_0}\right) = \frac{C_0}{S_0} - \frac{(1+c)C_0}{(1+s)S_0} = w_0 - \frac{(1+c)}{(1+s)}.w_0 \\
\Delta om &= \frac{(s-c)}{(1+s)}.w_0 = \frac{(s-c)}{(1+s)}.(1 - om_0) \\
\left(\frac{\Delta om}{om_0}\right) &= \frac{(s-c)}{(1+s)}.(\frac{1}{om_0} - 1)
\end{aligned} \tag{16}$$

Next, we derive the profit growth as a function of sales growth and percentage change of operating profit margin ratio.

$$NI_1 = (om_1).S_1 = (om_0 + \Delta om).(1+s).S_0$$
$$NI_0 = (om_0).S_0$$
$$g = \frac{NI_1}{NI_0} - 1 = \frac{(om_0 + \Delta om).(1+s).S_0}{(om_0).S_0} - 1 = \left(1 + \frac{\Delta om}{om}\right).(1+s) - 1 \quad (17)$$
$$g = s + (1+s).\left(\frac{\Delta om}{om_0}\right)$$

Then, we link the above profit growth equation with the percentage change in operating profit margin ratio equation to get the following.

$$g = s + (1+s).\frac{(s-c)}{(1+s)}.\left(\frac{1}{om_0} - 1\right) \quad (18)$$

In words, the equation 17 clearly shows us that the profit growth depends on two factors. The first factor is the sales growth whereas the second factor is the interaction between sales growth and the percentage change in operating profit margin ratio. It reveals that given the same sales growth, a firm with a higher percentage change in operating profit margin ratio would have a higher growth. We assume that the operating profit margin ratio (om) is positive. Then, equation 16 points out that given the same sales and cost growth rates, the percentage change in operating profit margin ratio would be higher for a less profitable firm (lower om_0). As a result, a less profitable firm would have a higher profit growth rate. Intuitively, a less profitable firm has such a low operating profit margin ratio such that when sales growth is higher than cost growth and the margin starts to improve, it counts as a larger percentage increase. This results in a higher growth rate in profit when compared to a more profitable firm.

Another conclusion we can draw from the formulae in equation (14) and (16) is that if sales growth and cost growth are the same then profit growth will also equal to sales growth and the margin remains constant. However, if sales growth is higher (lower) than cost growth, then profit growth will be even higher (lower) than sales growth and the operating profit margin ratio will increase (decrease) accordingly.

In addition, there is also a relationship between a change in operating margin ratio (om), sales growth (s) and fixed cost as a proportion of sales ratio (fc). Arellano (2007) derives this relationship by assuming a constant fixed cost and a constant variable cost ratio. We show his derivation below by beginning with the definition of operating profit margin ratio.

$$om_0 = \frac{\pi_0}{S_0} = \frac{(S_0 - VC_0 - FC)}{S_0}$$
$$om_1 = \frac{\pi_1}{S_1} = \frac{(S_1 - VC_1 - FC)}{S_1} \quad (19)$$

If a variable cost ratio (VC/S) is constant, then the growth rate of variable cost will be the same as sales growth.

$$\begin{aligned} & S_1 = S_0(1+s),\ VC_1 = VC_0(1+s) \\ & om_1 = \frac{S_1 - VC_1 - FC}{S_1} = \frac{S_0(1+s) - VC_0(1+s) - FC}{S_0(1+s)} \\ & FC = S_0 - VC_0 - \pi_0 \end{aligned} \tag{20}$$

We substitute fixed costs (FC) in equation (20) into the operating profit margin ratio equation (om_1).

$$\begin{aligned} om_1 &= \frac{S_0.s - VC_0.s + \pi_0}{S_0(1+s)} = \frac{s - vc.s + om_0}{(1+s)} = \frac{om_0 + s.(1-vc)}{(1+s)} \\ &= \frac{om_0 + s.(om_0 + fc_0)}{(1+s)} = \frac{om_0(1+s) + s.fc_0}{(1+s)} \\ om_1 &= om_0 + \frac{s}{(1+s)}.fc_0 \\ \Delta om &= \frac{s}{(1+s)}.fc_0 \end{aligned} \tag{21}$$

Arellano (2007) shows that if we know total fixed cost, then we can forecast a change in operating profit margin ratio, given the sales growth rate. This paper extends his approach further. We argue that instead of using the above equation in financial planning we can apply it in terms of financial statement analysis. We invert the equation to get the following relationship.

$$\begin{aligned} \Delta om &= \frac{s}{(1+s)}.fc_0 \\ fc_0 &= \frac{(1+s)}{s}.\Delta om \\ fc_1 &= \frac{FC}{S_1} = \frac{1}{(1+s)}.\frac{FC}{S_0} = \frac{1}{(1+s)}.fc_0 \end{aligned} \tag{22}$$

$$fc_1 = \frac{1}{s}.\Delta om \tag{23}$$

The above equation (23) reveals that we can find the current fixed cost ratio (fc_1 = FC/S_1) quite easily by dividing a change in operating profit margin ratio by the sales growth rate. Basically, we can infer management accounting information simply from reported financial statements. However, implicitly here, we assume a constant total fixed cost (FC) and a constant variable cost ratio (vc).

Furthermore, we can divide the following accounting identity by sales to get a relationship between operating profit margin ratio, variable cost ratio, and fixed cost ratio.

$$\begin{aligned} \text{Sales} &= \text{Profit} + \text{Variable Cost} + \text{Fixed Cost} \\ 1 &= om_1 + vc + fc_1 \\ vc &= 1 - om_1 - fc_1 \end{aligned} \tag{24}$$

Since we know fixed cost ratio from a change in operating profit margin ratio in equation (20), we can calculate the variable cost ratio (vc = VC_1/S_1) easily.

Furthermore, we can calculate the contribution margin (cm), defined as profit above variable cost, and then the sales breakeven as a proportion of sales.

$$\begin{aligned} cm &= 1 - vc \\ vc &= 1 - om_1 - fc_1 \\ cm &= om_1 + fc_1 = om_1 + \frac{1}{s}.\Delta om \end{aligned} \tag{25}$$

$$\begin{aligned} S_{BE} &= \frac{FC}{cm} \\ \frac{S_{BE}}{S_1} &= \frac{FC/S_1}{cm} = \frac{fc_1}{om_1 + fc_1} = \frac{\Delta om}{s \times om_1 + \Delta om} \end{aligned} \tag{26}$$

$$\text{Note: } fc_1 = \frac{1}{s}.\Delta om$$

The above formula shows that the higher the operating profit margin ratio, the lower the breakeven sales. In contrast, the higher the fixed cost ratio, the higher the sales breakeven.

3.1.2 Numerical examples

To illustrate the technique above, we apply them on the information in the table 1 below. The table 1 provides information on sales, cost, and operating profit margin. In this example, we assume that the only variable cost (VC) is the cost of goods sold (COGS) and the only fixed cost (FC) is the selling and general administrative expenses (SGA).

Table 1 Sales, Cost of Goods Sold (COGS) and Operating Profit Margin

	Year 0	Year 1	Δ (%Δ)
Sales	100	130	30 (30%)
COGS (VC)	(80)	(104)	24 (30%)
SGA (FC)	(10)	(10)	0 (0%)
Total Cost (TC)	(90)	(114)	24 (26.7%)
EBIT	10	16	6 (60%)

Note: s = 30%, c = 26.7%, g = 60%, w_0 = 90/100 = 90%, w_1 = 114/130 = 87.7%, om_0 = 1 – 90% = 10%, om_1 = 1 – 87.7% = 12.3%, vc = 80/100 = 104/130 = 80%, fc_0 = 10/100 = 10%, fc_1 = 10/130 = 7.7% Our formulae relate sales growth, cost growth, profit growth and its associated margins. If we know any three of these variables, we can then deduce the rest. We begin with profit growth and use the initial year (period 0) information.

$$g=\frac{(s\ -\ w_0.c)}{(1\ -\ w_0)}\ =\ \frac{(30\%\ -\ 0.9\times 26.7\%)}{(1\ -\ 0.9)}\ =\ \frac{6\%}{0.1}=60\%$$

$$=\frac{s-(1-om_0).c}{om_0}=\frac{30\%-(1-0.1)\times 26.7\%}{0.1}=\frac{6\%}{0.1}=60\%$$

$$g=s+\frac{w_0}{(1-w_0)}.(s-c)=30\%+\frac{0.9}{(1-0.9)}.(30\%-26.7\%)=30\%+30\%=60\%$$

$$=s+\frac{(1-om_0)}{om_0}.(s-c)=30\%+\frac{(1-0.1)}{0.1}.(30\%-26.7\%)=30\%+30\%=60\%$$

$$=c+\frac{1}{(1-w_0)}.(s-c)=26.7\%+\frac{1}{(1-0.9)}.(30\%-26.7\%)=26.7\%+33.3\%=60\%$$

$$=c+\frac{1}{om_0}.(s-c)=26.7\%+\frac{1}{0.1}.(30\%-26.7\%)=26.7\%+33.3\%=60\%$$

We can infer the initial cost ratio (w_0) and implicitly the initial operating profit margin ratio (om_0) from growth rates of sales, cost, and profit. In addition, if we know the growth rate, then we can update the initial cost ratio (w_0) to the current cost ratio (w_1) and vice versa.

$$w_0\ =\ \frac{(g\ -\ s)}{(g\ -\ c)}=\frac{(60\%\ -\ 30\%)}{(60\%\ -\ 26.7\%)}=\frac{30\%}{33.3\%}=0.9=90\%$$

$$om_0=1-w_0=1-90\%=10\%$$

$$w_1\ =\ w_0\frac{(1+c)}{(1+s)}=90\%\frac{(1\ +\ 26.7\%)}{(1\ +\ 30\%)}=87.7\%$$

$$om_1=1-w_1=1-87.7\%=12.3\%$$

$$w_0\ =\ w_1\frac{(1+s)}{(1+c)}=87.7\%\frac{(1\ +\ 30\%)}{(1\ +\ 26.7\%)}=90\%$$

In practical applications, however, we are normally more interested in the current cost ratio (w_1) or the current operating profit margin ratio ($om_1 = 1 - w_1$) than the initial ones. Still, we could deduce profit growth and cost growth.

$$g=\frac{\left[\dfrac{(s-w_1.c)}{(1-w_1)}\right]+(s.c)}{\left[1-\dfrac{(w_1.s-c)}{(1-w_1)}\right]}=\frac{\left[\dfrac{0.30-(0.877)(0.267)}{1-0.877}\right]+(0.30)(0.267)}{\left[1-\dfrac{(0.877)(0.30)-0.267}{1-0.877}\right]}=\frac{0.62}{1.032}=60\%$$

$$c=\frac{\left(g+\dfrac{1}{w_1}\dfrac{(s-g)}{(1+s)}\right)}{\left(1-\dfrac{1}{w_1}\dfrac{(s-g)}{(1+s)}\right)}=\frac{\left(0.6+\dfrac{1}{0.877}\dfrac{(0.3-0.6)}{(1+0.3)}\right)}{\left(1-\dfrac{1}{0.877}\dfrac{(0.3-0.6)}{(1+0.3)}\right)}=\frac{0.34}{1.26}=26.7\%$$

If we assume a constant total fixed cost (FC) and a constant variable cost ratio (vc), we can also infer the fixed cost ratio (fc) and the variable cost ratio (vc) just from a change in operating profit margin ratio and sales growth.

$$om_1 = om_0 + \frac{s}{(1+s)}.fc_0 = 10\% + \frac{0.3}{(1+0.3)} \times 10\% = 10\% + 2.3\% = 12.3\%$$

$$\Delta om = \frac{s}{(1+s)}.fc_0 = \frac{0.3}{(1+0.3)} \times 10\% = 2.3\%$$

$$fc_0 = \frac{(1+s)}{s}.\Delta om = \frac{(1+0.3)}{0.3} \times 2.3\% = 10\%$$

In practice, we focus more on current figures.

$$fc_1 = \frac{1}{s}.\Delta om = \frac{1}{0.3} \times 2.3\% = 7.7\%$$

$$vc = 1 - om_0 - fc_0 = 1 - 10\% - 10\% = 80\%$$

$$= 1 - om_1 - fc_1 = 1 - 12.3\% - 7.7\% = 80\%$$

$$cm = 1 - vc = 1 - 80\% = 20\%$$

Since we deduce the contribution margin (cm) and the current fixed cost ratio (fc_1), we could easily compute the current break-even sales.

$$\frac{S_{BE}}{S_1} = \frac{fc_1}{om_1 + fc_1} = \frac{7.7\%}{12.3\% + 7.7\%} = \frac{7.7\%}{20\%} = 0.385 = 38.5\%$$

$$= \frac{\Delta om}{s \times om_1 + \Delta om} = \frac{2.3\%}{(0.3) \times 12.3\% + 2.3\%} = 38.5\%$$

This simply means that sales need to drop more than 61.5% (1 – 38.5%) for this firm to face a loss.

3.2 Relationships among contribution margin ratio (cm), variable cost ratio (vc), fixed cost ratio (fc), operating margin ratio (om), degree of operating leverage (DOL) and sales breakeven (S_{BE})

3.2.1 Theoretical models

In this part, we assume that total fixed cost and variable cost ratio (total variable cost as a proportion of sales) remain constant. We begin with the definition and derivation of DOL as shown in standard textbooks [e.g. Koh et al. (2014), Maher et al. (2011)]. The degree of operating leverage (DOL) is defined as percentage change of earnings before interest and taxes (EBIT) per one percentage change of sales. It is mainly determined by the cost structure. More specifically, a firm with a higher fixed cost ratio would have a higher DOL. The formulae are restated below.

$$S_{BE} = \frac{FC}{cm}$$

$$DOL_S = \frac{\%\Delta \text{EBIT}}{\%\Delta S} = \frac{\frac{\Delta EBIT}{EBIT}}{\frac{\Delta S}{S}} = \frac{S - VC}{S - VC - FC} = \frac{S}{S - S_{BE}} \tag{1}$$

$$= \frac{EBIT + FC}{EBIT} = \frac{\text{Contribution Margin}}{EBIT} = \frac{\text{Contribution Margin Ratio}}{\text{Operating Profit Margin Ratio}} = \frac{\text{cm}}{\text{om}}$$

The derivation is shown below (Guidry, Horrigan, & Craycraft, 1998; Prezas, 1987; Zivney, 2000). There are two types of costs, namely, variable cost (VC) and fixed cost (FC). The variable “vc” stands for variable cost ratio (VC/S).

$$DOL = \frac{\%\Delta EBIT}{\%\Delta S} = \frac{\frac{(EBIT_1 - EBIT_0)}{EBIT_0}}{\frac{(S_1 - S_0)}{S_0}}$$

$$= \frac{\left(\frac{(S_1 - VC_1 - FC) - (S_0 - VC_0 - FC)}{S_0 - VC_0 - FC}\right)}{\frac{(S_1 - S_0)}{S_0}} = \frac{(S_1 - S_0) - (VC_1 - VC_0)}{(S_0 - VC_0 - FC_0)} \cdot \frac{S_0}{(S_1 - S_0)}$$

$$= \frac{\Delta S - \Delta VC}{(S_0 - VC_0 - FC)} \cdot \frac{S_0}{\Delta S} = \frac{(1 - vc) \times \Delta S}{(S_0 - VC_0 - FC)} \cdot \frac{S_0}{\Delta S} = \frac{(1 - vc) \times S_0}{(S_0 - VC_0 - FC)} = \frac{(S - VC_0)}{(S_0 - VC_0 - FC)}$$

$$DOL = \frac{(S_0 - VC_0)}{(S_0 - VC_0 - FC)} = \frac{EBIT_0 + FC}{EBIT_0}$$

$$= \frac{(1 - VC) \times S_0}{(S_0 - VC_0 - FC)} = \frac{(1 - VC) \times S_0}{(1 - VC) \times S_0 - FC} = \frac{S_0}{\left(S_0 - \frac{FC}{(1 - VC)}\right)}$$

$$DOL = \frac{S_0}{(S_0 - S_{BE})}$$

The variable “S_{BE}” stands for sales breakeven. It is simply the fixed cost (FC) divided by the contribution margin or one minus the variable cost ratio (1 – VC). Simply put, DOL would equal to sales divided by sales above the breakeven.

Basically, DOL is the ratio of contribution margin ratio (cm) and operating margin ratio (om). Now, we can start to link the definition of DOL, break even, variable cost ratio (vc) and fixed cost ratio (fc).

$$DOL_0 = \frac{\%\Delta \text{EBIT}}{\%\Delta S} = \frac{g}{s} = \frac{cm}{om_0} \tag{27}$$

$$cm = DOL_0 \times om_0 = \left(\frac{g}{s}\right) \times om_0 \tag{28}$$

$$vc = 1 - cm = 1 - DOL_0 \times om_0 = 1 - \left(\frac{g}{s}\right) \times om_0 \tag{29}$$

Since the contribution margin (CM) is simply sales minus variable cost [CM = Sales (S) – Variable Cost (VC)], the variable cost ratio (vc = VC/S) would simply be one minus the contribution margin ratio (cm). Then, we plug-in the formula for DOL.

We can get the estimate of DOL easily as the ratio between profit growth (g) over sales growth (s). The operating margin is simply the ratio of EBIT over sales and we can get this information quite easily from reported financial statements. By using the above formulae, we can then estimate the contribution margin ratio (cm) and variable cost ratio (vc) quite quickly.

Because the operating margin (OM) is contribution margin (CM) minus fixed cost (FC), we can simply restate fixed cost ratio (fc) as simply contribution margin ratio (cm) minus operating profit margin ratio (om).

$$OM = CM - FC \Rightarrow \frac{OM}{S_0} = \frac{CM}{S_0} - \frac{FC}{S_0} \Rightarrow om_0 = cm - fc_0 \qquad (30)$$

$$fc_0 = cm - om_0 = (DOL_0 - 1) \times om_0 = \left(\frac{g}{s} - 1\right) \times om_0 \qquad (31)$$

Furthermore, we can use the definition of DOL and the DOL formula to solve for the sales breakeven as a proportion of sales.

$$\begin{aligned} DOL_0 &= \frac{\%\Delta \text{EBIT}}{\%\Delta S} = \frac{g}{s} = \frac{S_0}{S_0 - S_{BE}} \\ \frac{S_{BE}}{S_0} &= \frac{(DOL_0 - 1)}{DOL_0} = \frac{(g-s)}{g} \end{aligned} \qquad (32)$$

However, the DOL above is that of the initial period (period 0). In order to practically apply the formula, we need to update DOL from period 0 to period 1. We derive the updating relationship below. The variable “E” stands for EBIT.

$$\begin{aligned} DOL_0 &= \frac{g}{s} = \frac{E_0 + FC}{E_0} = \left(1 + \frac{FC}{E_0}\right) \\ FC &= \left(\frac{g-s}{s}\right).E_0 \\ fc_0 &= (DOL_0 - 1) \times om_0 = \left(\frac{g-s}{s}\right) \times om_0 \\ DOL_1 &= \frac{E_1 + FC}{E_1} = \frac{(1+g).E_0 + FC}{(1+g).E_0} = \frac{(1+g).E_0 + \left(\frac{g-s}{s}\right).E_0}{(1+g).E_0} \\ &= \frac{s + s.g + g - s}{s.(1+g)} = \frac{g}{s}.\frac{(1+s)}{(1+g)} \\ DOL_1 &= \frac{g}{s}.\frac{(1+s)}{(1+g)} = DOL_0.\frac{(1+s)}{(1+g)} \end{aligned} \qquad (33)$$

Similarly, we need to update the fixed cost ratio (fc) and sales breakeven (S_{BE}) to reflect current values.

$$fc_1 = \frac{FC}{S_1} = \frac{FC}{S_0(1+s)} = \frac{1}{(1+s)}.\frac{FC}{S_0} = \frac{1}{(1+s)}.fc_0 = \frac{1}{(1+s)}.\left(\frac{g-s}{s}\right) \times om_0$$

$$om_1 = om_0.\frac{(1+g)}{(1+s)} \Leftrightarrow om_0 = om_1.\frac{(1+s)}{(1+g)} \tag{34}$$

$$fc_1 = \frac{1}{(1+g)}.\left(\frac{g-s}{s}\right) \times om_1 \tag{35}$$

$$\frac{S_{BE}}{S_0} = \frac{(DOL_0 - 1)}{DOL_0} = \frac{(g-s)}{g}$$

$$\frac{S_{BE}}{S_1} = \frac{1}{(1+s)}.\frac{S_{BE}}{S_0} = \frac{1}{(1+s)}.\frac{(g-s)}{g} = [DOL_1/(1+s) - 1/(1+g)]/DOL_1 \tag{36}$$

There is no updating for the level of fixed cost (FC), the variable cost ratio (vc) or the contribution margin ratio (cm) because we implicitly assume that they are constant. The implication is that the level of sales breakeven (S_{BE}) will also remain constant.

3.2.2 Numerical examples

Again, to illustrate the technique above, we apply them on the information in table 1.

If the assumption of a constant total fixed cost, (FC) and a constant variable cost ratio (vc) hold then any DOL formula would give the same correct number.

$$DOL_0 = \frac{\%\Delta EBIT}{\%\Delta S} = \frac{60\%}{30\%} = 2$$

$$DOL_0 = \frac{(EBIT_0 + FC)}{EBIT_0} = \frac{(10+10)}{10} = 2$$

$$S_{BE,0} = \frac{FC_0}{(1-vc)} = \frac{10}{(1-0.8)} = 50$$

$$DOL_0 = \frac{S_0}{(S_0 - S_{BE,0})} = \frac{100}{(100-50)} = 2$$

However, the DOL above is that of the initial period (period 0). In practice, we are more interested in the current DOL (period 1). We need to update DOL from period 0 to period 1.

$$S_{BE,1} = \frac{FC}{(1-vc)} = \frac{10}{(1-0.8)} = 50$$

$$DOL_1 = \frac{S_1}{(S_1 - S_{BE,1})} = \frac{130}{(130-50)} = 1.625$$

$$DOL_1 = DOL_0.\frac{(1+s)}{(1+g)} = 2 \times \frac{(1+0.3)}{(1+0.6)} = 1.625$$

We can then compute the contribution margin ratio (cm), the variable cost ratio (vc), the fixed cost ratio (fc) and sales breakeven just from growth rates and operating margin ratio (om).

$$DOL_0 = \frac{g}{s} = \frac{60\%}{30\%} = 2$$

$$cm = DOL_0 \times om_0 = 2 \times 10\% = 20\%$$

$$cm = \frac{(S - VC)}{S} = \frac{(100 - 80)}{100} = 20\%$$

$$vc = 1 - cm = 1 - 0.20 = 80\%$$

$$vc = \frac{VC_0}{S_0} = \frac{VC_1}{S_1} = \frac{80}{100} = \frac{104}{130} = 80\%$$

$$fc_0 = (DOL - 1) \times om_0 = (2 - 1) \times 10\% = 10\%$$

$$fc_0 = \frac{FC}{S_0} = \frac{10}{100} = 10\%$$

$$fc_0 = \left(\frac{g - s}{s}\right) \times om_0 = \left(\frac{60\% - 30\%}{30\%}\right) \times 10\% = 10\%$$

$$fc_1 = \frac{FC}{S_1} = \frac{10}{130} = 7.7\%$$

$$fc_1 = \frac{1}{(1+s)}.fc_0 = \frac{1}{(1+0.3)} \times 10\% = 7.7\%$$

$$\frac{S_{BE}}{S_0} = \frac{(DOL_0 - 1)}{DOL_0} = \frac{(g - s)}{g} = \frac{(2-1)}{2} = \frac{(60\% - 30\%)}{60\%} = 50\%$$

$$\frac{S_{BE}}{S_1} = \frac{1}{(1+s)}.\frac{S_{BE}}{S_0} = \frac{1}{(1+0.3)} \times 50\% = 38.5\% = [DOL_1/(1+s) - 1/(1+g)]/DOL_1$$
$$= [1.625/(1+0.3) - 1/(1+0.6)]/1.625$$

This simply means that sales need to drop more than 61.5% (1 – 38.5%) for this firm to face a loss.

3.3 Case of a known change in total fixed cost and variable cost ratio

3.3.1 Theoretical models

This part relaxes the above assumptions and extends our approach to the case of a known change in total fixed cost (FC) and variable cost ratio (vc). In application, these changes could be approximated from reported financial statements and industry price index series.

We begin our analysis with a total variable cost (VC). If the variable cost ratio (vc = VC/S) remains stable, then the variable cost growth rate will be the same as that of the sales growth rate (s). However, if the variable cost ratio changes, then the growth rate of variable cost will be the interaction between sales growth (s) and a percentage change of the variable cost ratio (v). Mathematically, it can be stated in the following equation.

$$VC_1 = (1+s)(1+v).VC_0 \tag{37}$$

In practice, we estimate "v", percentage change in the variable cost ratio, by using industry price index series. Let "p" and "x" be the percentage change in the output price and variable input price, respectively. For example, in the chicken industry, "p" is the percentage change in the price of chicken and "x" then is the percentage change in the price of chicken feed. We state this relationship in the following equation.

$$(1+v) = \frac{(1+x)}{(1+p)} \tag{38}$$

Mathematically, we can state EBITs (E) of the initial year and the current year in the following equations.

$$\begin{array}{lccccccccccc} \text{Year 0:} & S_0 & & -FC_0 & & -VC_0 & = & S_0 & & -C_0 & = & E_0 \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{Year 1:} & (1+s)S_0 & - & (FC_0+\Delta FC) & - & (1+s)(1+v)VC_0 & = & (1+s)S_0 & - & (1+c)C_0 & = & (1+g)E_0 \end{array}$$

Similarly, we can state total costs (C) and its change in the following equations.

$$\begin{aligned} C_0 &= FC_0 + VC_0 \\ C_1 &= FC_0 + \Delta FC + (1+s)(1+v)\times VC_0 \\ \Delta C &= \Delta FC + [(1+s)(1+v)-1]\times VC_0 \end{aligned} \tag{39}$$

We can solve for "VC_0" and then divide through with the initial sales (S_0) to get the initial variable cost ratio (vc_0).

$$\begin{aligned} FC_0 &= C_0 - VC_0 && \Rightarrow fc_0 = w_0 - vc_0 \\ VC_0 &= \frac{(\Delta C - \Delta FC)}{[(1+s)(1+v)-1]} && \Rightarrow vc_0 = \frac{(w_0.c - \Delta fc)}{[(1+s)(1+v)-1]} \\ \frac{\Delta C}{S_0} &= \frac{C_0}{S_0}.\frac{\Delta C}{C_0} = w_0.c \\ \frac{\Delta FC}{S_0} &= \Delta fc \end{aligned} \tag{40}$$

Next, we update our analysis to the current year.

$$vc_0 = \frac{(w_0.c - \Delta fc)}{[(1+s)(1+v)-1]}$$

$$fc_0 = w_0 - vc_0$$

$$VC_1 = (1+s)(1+v).VC_0$$

$$FC_1 = FC_0 + \Delta FC$$

$$\frac{VC_1}{S_0}=(1+s)(1+v).\frac{VC_0}{S_0}=\left(\frac{(1+s)(1+v)}{[(1+s)(1+v)-1]}\right).(w_0.c-\Delta fc) \quad (41)$$

$$\frac{FC_1}{S_0}=\frac{FC_0}{S_0}+\frac{\Delta FC}{S_0}=fc_0+\Delta fc=(w_0-vc_0)+\Delta fc \quad (42)$$

Then, we need to change the denominator from the initial sales (S_0) to the current sales (S_1) to complete our update.

$$vc_1=\frac{VC_1}{S_1}=\frac{VC_1}{(1+s)S_0}=\left(\frac{(1+s)(1+v)}{[(1+s)(1+v)-1]}\right).\frac{(w_0.c-\Delta fc)}{(1+s)} \quad (43)$$

$$fc_1=\frac{FC_1}{S_1}=\frac{FC_1}{(1+s)S_0}=\frac{(w_0-vc_0)+\Delta fc}{(1+s)} \quad (44)$$

After we get the variable cost ratio (vc) and the fixed cost ratio (fc), then we can calculate DOL and sales breakeven quite easily.

$$cm=1-vc$$

$$om=1-vc-fc$$

$$DOL_0=\frac{cm_0}{om_0},DOL_1=\frac{cm_1}{om_1}$$

$$\frac{S_{BE,0}}{S_0}=\frac{fc_0}{cm_0},\frac{S_{BE,1}}{S_1}=\frac{fc_1}{cm_1}$$

The numerical example in the next section would illustrate the applications.

In addition, we could also extend Arellano (2007)'s model to this case of a known change in total fixed cost and variable cost ratio. We begin by restating the operating profit margin ratio (om).

$$vc_1=\left(\frac{(1+s)(1+v)}{[(1+s)(1+v)-1]}\right).\frac{(w_0.c-\Delta fc)}{(1+s)} \quad (43)$$

$$fc_1=\frac{(w_0-vc_0)+\Delta fc}{(1+s)} \quad (44)$$

$$om_1=1-vc_1-fc_1=1-\left(\frac{(1+s)(1+v)}{[(1+s)(1+v)-1]}\right).\frac{(w_0.c-\Delta fc)}{(1+s)}-\left(\frac{(w_0-vc_0)+\Delta fc}{(1+s)}\right)$$

$$vc_0=\frac{(w_0.c-\Delta fc)}{[(1+s)(1+v)-1]}$$

$$fc_0=w_0-vc_0$$

$$om_0=1-vc_0-fc_0=1-\frac{(w_0.c-\Delta fc)}{[(1+s)(1+v)-1]}-(w_0-vc_0)$$

Then, we compute the change in operating profit margin ratio (Δom).

$$om_1 - om_0 = \left[1 - \left(\frac{(1+s)(1+v)}{[(1+s)(1+v)-1]}\right).\frac{(w_0.c - \Delta fc)}{(1+s)} - \left(\frac{(w_0 - vc_0) + \Delta fc}{(1+s)}\right)\right] - \left[1 - \frac{(w_0.c - \Delta fc)}{[(1+s)(1+v)-1]} - (w_0 - vc_0)\right]$$

$$\Delta om = -v.\left(\frac{(w_0.c - \Delta fc)}{[(1+s)(1+v)-1]}\right) + \left(\frac{[s.(w_0 - vc_0) - \Delta fc]}{(1+s)}\right) \tag{45}$$

$$\Delta om = -v.\left(\frac{(w_0.c - \Delta fc)}{[(1+s)(1+v)-1]}\right) + \left(\frac{[s.fc_0 - \Delta fc]}{(1+s)}\right)$$

Next, we solve for the initial fixed cost ratio (fc_0).

$$fc_0 = \frac{1}{s}.\left[(1+s).\left(\Delta om + v.\left(\frac{(w_0.c - \Delta fc)}{[(1+s)(1+v)-1]}\right)\right) + \Delta fc\right] \tag{46}$$

We rearrange terms to update the fixed cost ratio to the current value.

$$FC_1 = FC_0 + \Delta FC$$
$$\frac{FC_1}{S_1} = \frac{fc_0 \times S_0}{S_1} + \frac{\Delta FC}{S_0}.\frac{S_0}{S_1}$$
$$fc_1 = \frac{fc_0}{(1+s)} + \frac{\Delta fc}{(1+s)} \tag{47}$$
$$fc_1 = \frac{1}{(1+s)}.(fc_0 + \Delta fc)$$

If there are no changes in the variable cost ratio and the total fixed cost (v = 0 and Δfc = 0), then we would get Arellano (2007)'s results. We restate the results below.

$$\Delta om = \frac{s}{(1+s)}.fc_0$$
$$fc_0 = \frac{(1+s)}{s}.\Delta om$$
$$fc_1 = \frac{1}{s}.\Delta om$$

3.3.2 Numerical examples

To illustrate the technique above, we apply them on the information in table 2 below. The table 2 provides information on sales, cost, and operating profit margin. In this example, we assume a known change in total fixed cost and variable cost ratio. The known change in total fixed cost is ten (ΔFC = 10). The percentage change in input price is five percent (x = 5%), whereas the percentage change in output price is two percent (p = 2%). Therefore, the implied percentage change in the variable cost ratio (v) is 2.94% based on the following equation.

$$(1+v) = \frac{(1+x)}{(1+p)} = \frac{(1+5\%)}{(1+2\%)} = \frac{1.05}{1.02} = 1.0294$$
$$v = 2.94\%$$

The percentage change in the total variable cost is simply the interaction between sales growth and percentage change in the variable cost ratio as shown in the equation below.

$$\left(1+\frac{\Delta VC}{VC}\right)=(1+s)(1+v)=(1+30\%)(1+2.94\%)=1.3382$$

$$\frac{\Delta VC}{VC}=1.3382-1=33.82\%$$

Table 2 Sales, Cost of Goods Sold (COGS) and Operating Profit

	Year 0	Year 1	△ (%△)
Sales	100	130.00	30 (30%)
COGS (VC)	(80)	(107.06)	27.06 (33.82%)
SGA (FC)	(10)	(20.00)	10 (100%)
Total Cost (TC)	(90)	(127.06)	37.06 (41.18%)
EBIT	10	2.94	-7.06 (-70.59%)

Note: s = 30%, c = 41.18%, g = -70.59%, w_0 = 90/100 = 90%, w_1 = 127.06/130 = 97.74%, om_0 = 1 – 90% = 10%, om_1 = 1 – 97.74% = 2.26%, vc_0 = 80/100 = 80%, vc_1 = 107.06/130 = 82.35%, fc_0 = 10/100 = 10%, fc_1 = 20/130 = 15.38%, v = 2.94%, ΔFC = 10, Δfc = ΔFC/S_0 = 10/100 = 0.1

We begin with the initial variable cost ratio (vc_0) and fixed cost ratio (fc_0). The only data we need are the cost ratio (w_0), the change in fixed cost as a proportion of the initial sales (Δfc), sales growth (s), cost growth (c), and the percentage change in the variable cost ratio (v).

$$vc_0=\frac{(w_0.c-\Delta fc)}{[(1+s)(1+v)-1]}=\frac{(0.9\times 0.4118-0.1)}{[1.3382-1]}=0.8=80\%$$

$$cm_0=1-vc_0=1-0.8=20\%$$

$$fc_0=w_0-vc_0=0.9-0.8=0.1=10\%$$

$$om_0=1-vc_0-fc_0=1-80\%-10\%=10\%$$

Then, we continue with the current variable cost ratio (vc_1) and fixed cost ratio (fc_1).

$$vc_1=\left(\frac{(1+s)(1+v)}{[(1+s)(1+v)-1]}\right).\frac{(w_0.c-\Delta fc)}{(1+s)}=\left(\frac{1.3382}{1.3382-1}\right).\frac{(0.9\times 0.4118-0.1)}{(1+0.3)}=82.35\%$$

$$cm_1=1-vc_1=1-82.35\%=17.65\%$$

$$fc_1=\frac{(w_0-vc_0+\Delta fc)}{(1+s)}=\frac{(0.9-0.8+0.1)}{(1+0.3)}=15.38\%$$

$$om_1=1-vc_1-fc_1=1-82.35\%-15.38\%=2.26\%$$

After we get the contribution margins (cm) and the fixed cost ratios (fc), we can easily compute DOLs and sales break evens.

$$DOL_0 = \frac{cm_0}{om_0} = \frac{20\%}{10\%} = 2$$
$$\frac{S_{BE,0}}{S_0} = \frac{fc_0}{cm_0} = \frac{10\%}{20\%} = 0.5 = 50\%$$

$$DOL_1 = \frac{cm_1}{om_1} = \frac{17.65\%}{2.26\%} = 7.81$$
$$\frac{S_{BE,1}}{S_1} = \frac{fc_1}{cm_1} = \frac{15.38\%}{17.65\%} = 0.8714 = 87.14\%$$

In practice, it is difficult to separate between a total fixed cost and a total cost from reported financial statements. That is why it is normally difficult to calculate a contribution margin (cm), a DOL and a sales breakeven if we do not have the company's internal management accounting data. The technique here lessens the information requirement. The only required input here is the change in fixed cost over a year and not the total fixed cost itself.

Practically, we could estimate this one from accounting items that we feel certain are fixed costs such as depreciation and amortization expenses reported in the cash flow statement. Other good examples are rental expense and administrative expense. We could calculate the changes of these expenses and use them as an approximation of the change in total fixed cost. Then, we simply calculate it as a proportion of the initial sales to get "Δfc" to be used in our formulae.

A percentage change in the variable cost ratio (v) could be approximated from percentage changes of reported industry price indices. For example, if this company's main product is chicken and the key input is chicken feed, then we can use the chicken price index series to calculate "p" and the chicken feed price index series to calculate "x" in the equation (38) to approximate "v".

In the real world, a firm produces many outputs and uses many inputs. To approximate "p" and "x" would not be as easy as in the single input and single output case. However, we can still approximate them by using weighted average of percentage changes in key input and key output prices. The weight for each output could be the revenue share of each product segment, whereas the weight for each input could be the cost share of each key input.

Of course, the numbers we get here would not be as accurate as the company's internal data. Nevertheless, we can estimate all these quite easily from just reported financial statements and industry price series. As such, the technique is useful in performing financial statement analysis.

4. Conclusion

This paper provides additional analytical tools that can be used in conjunction with traditional financial statement analysis technique like common size analysis, trend analysis and financial ratios (Wahlen, Baginski, & Bradshaw, 2017). More specifically, we provide links between growth rates of sales, cost, and profit and its associated margins. In terms of application, if we know any three of these variables, we can then deduce the rest. This is especially useful when we analyze financial highlight data where we do not have full financial statements. It also reveals that if sales growth is higher (lower) than cost growth, then a firm with a higher operating profit margin ratio will have a lower (higher) profit growth.[3]

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[3] Under the condition that the operating profit margin ratio is positive, and the sales growth rate and the cost growth rate are the same for both firms.

Appendix: Formula Summary for Practitioners

Notations

0 = initial period, 1 = current period, s = sales growth, c = cost growth, g = operating profit growth, w = cost ratio = Total Costs/Sales, om = operating profit margin ratio = 1 – w, Δom = change in operating profit margin ratio between the initial period and the current period, vc = variable cost ratio = Variable Costs/Sales, fc = fixed cost ratio = Fixed Costs/Sales, cm = contribution margin ratio = 1 – vc, S_{BE} = Sales Breakeven (Sales where EBIT = 0), S_{BE}/S = Sales Breakeven as a proportion of sales, v = percentage change in the variable cost ratio (%Δvc), Δfc = change in fixed costs as a proportion of initial sales = Change in fixed costs/Sales of initial period ($\Delta FC/S_0$)

Formula Group 1 There is no assumption concerning fixed costs and variable costs.

$$w_1 = w_0\frac{(1+c)}{(1+s)} \quad (4) \qquad om_1 = 1-(1-om_0)\frac{(1+c)}{(1+s)} \quad (5) \qquad w_0 = \frac{(g-s)}{(g-c)} \quad (9) \qquad w_1 = \frac{(g-s)}{(g-c)}.\frac{(1+c)}{(1+s)} \quad (10)$$

$$g = \frac{s-(1-om_0).c}{om_0} = \frac{\left[\dfrac{s-(1-om_1).c}{om_1}\right]+(s.c)}{\left[1-\dfrac{(1-om_1).s-c}{om_1}\right]} \quad (11) \qquad g = s+\frac{(1-om_0)}{om_0}.(s-c) = c+\frac{1}{om_0}.(s-c) \quad (14,\ 15)$$

$$g = s+(1+s).\left(\frac{\Delta om}{om_0}\right) = s+(1+s).\frac{(s-c)}{(1+s)}.\left(\frac{1}{om_0}-1\right) \quad (17,\ 18)$$

$$s = om_0.g+(1-om_0).c = \frac{om_1.g+(1+g-om_1).c}{\left[1+(1-om_1).g+om_1.c\right]} \quad (7,\ 8)$$

$$c = g+\frac{1}{(1-om_0)}(s-g) = \frac{\left(g+\dfrac{1}{w_1}\dfrac{(s-g)}{(1+s)}\right)}{\left(1-\dfrac{1}{w_1}\dfrac{(s-g)}{(1+s)}\right)} = \frac{\left(g+\dfrac{1}{(1-om_1)}\dfrac{(s-g)}{(1+s)}\right)}{\left(1-\dfrac{1}{(1-om_1)}\dfrac{(s-g)}{(1+s)}\right)} \quad (12,\ 13)$$

$$\Delta om = om_1-om_0 = \frac{(s-c)}{(1+s)}.(1-om_0), \quad \left(\frac{\Delta om}{om_0}\right) = \frac{(s-c)}{(1+s)}.\left(\frac{1}{om_0}-1\right) \quad (16)$$

Formula Group 2 We assume a constant fixed cost and a constant variable cost ratio.

$$\Delta om = \frac{s}{(1+s)}.fc_0 \quad (21) \qquad fc_0 = \frac{(1+s)}{s}.\Delta om_0 = \left(\frac{g}{s}-1\right)\times om_0 \quad (22) \qquad fc_1 = \frac{1}{s}.\Delta om \quad (23)$$

$$\frac{S_{BE}}{S_0} = \frac{(g-s)}{g} \quad (32) \qquad \frac{S_{BE}}{S_1} = \frac{1}{(1+s)}.\frac{(g-s)}{g} = \frac{\Delta om}{s\times om_1+\Delta om} \quad (26,\ 36) \qquad DOL_0 = \frac{g}{s} \quad (27)$$

$$DOL_1 = \frac{g}{s}.\frac{(1+s)}{(1+g)} \quad (33) \qquad cm = DOL_0\times om_0 = \left(\frac{g}{s}\right)\times om_0 \quad (28) \qquad vc = 1-DOL_0\times om_0 = 1-\left(\frac{g}{s}\right)\times om_0 \quad (29)$$

Formula Group 3 We assume a known change in total fixed cost and a known percentage change in the variable cost ratio.

$$vc_0 = \frac{(w_0.c - \Delta fc)}{[(1+s)(1+v)-1]} \quad (40)$$

$$fc_0 = w_0 - vc_0 = \frac{1}{s}.\left[(1+s).\left(\Delta om + v.\left(\frac{(w_0.c - \Delta fc)}{[(1+s)(1+v)-1]}\right)\right) + \Delta fc\right] \quad (46)$$

$$cm_0 = 1 - vc_0,\ DOL_0 = \frac{cm_0}{om_0}, \frac{S_{BE,0}}{S_0} = \frac{fc_0}{cm_0}$$

$$vc_1 = \left(\frac{(1+s)(1+v)}{[(1+s)(1+v)-1]}\right).\frac{(w_0.c - \Delta fc)}{(1+s)} \quad (43)$$

$$fc_1 = \frac{(w_0 - vc_0) + \Delta fc}{(1+s)} = \frac{1}{(1+s)}.(fc_0 + \Delta fc) \quad (44)$$

$$cm_1 = 1 - vc_1, DOL_1 = \frac{cm_1}{om_1}, \frac{S_{BE,1}}{S_1} = \frac{fc_1}{cm_1}$$

$$\Delta om = -v.\left(\frac{(w_0.c - \Delta fc)}{[(1+s)(1+v)-1]}\right) + \left(\frac{[s.fc_0 - \Delta fc]}{(1+s)}\right) \quad (45)$$